Recursive Preferences, the Value of Life, and Household Finance*

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Abstract

We analyze lifecycle saving using a recursive utility model calibrated to match estimates of the value of a statistical life. The novelty of our approach is that we require preferences to be monotone with respect to first-order stochastic dominance while disentangling risk aversion and the intertemporal elasticity of substitution. We show that, with a positive value of life, risk aversion reduces each of savings, stock market participation, and annuity purchase. Risk averse agents insure against early death by consuming more when young and retaining wealth for bequests. These results contrast with those of previous studies using non-monotonic recursive models.

Keywords: lifecycle model, value of life, risk aversion, saving choices, portfolio choices, annuity puzzle, recursive utility.

JEL codes: D91, G11, J14, J17.

1 Introduction

"Nothing, they say is more certain than death, and nothing more uncertain than the time of dying" – Thomas Paine.

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The certainty of death, and the unpredictability of its timing, are fundamental features of human existence. Any analysis of an aspect of human behavior to which mortality is relevant must reflect on how people react to lifetime uncertainty and to how they respond to opportunities to modify their mortality risk. A coherent framework for investigating the interaction between human behavior and mortality risk is particularly relevant in a period where the risk of dying could change significantly due to medical progress, or due to the occurrence of novel risks.

Although there are some exceptions, the economic literature on such matters is mostly split in two branches, depending on whether mortality risk is considered to be exogenous or endogenous. On the one hand, papers in the Household Finance literature (hereafter HF) tackle questions related to consumption, saving and financial portfolio choices over the lifecycle. Mortality is usually considered to be exogenous, and as such, the willingness to pay for mortality risk reduction tends not to be considered.

On the other hand, the literature on the value of life (hereafter VoL) considers questions related to endogenous mortality risk reduction. This literature studies the trade-off between wealth and mortality, which is key to the evaluation of public policies aiming at lowering mortality risk – such as road safety investments, public health spending, or nonpharmaceutical interventions during a pandemic. Papers in this literature have paid limited attention to saving behaviors or portfolio choices and how they interact with mortality risk.

Both strands of literature were initiated with contributions relying on the same decision model: the standard additive expected utility model, as introduced by Yaari (1965) in the HF literature and used by Shepard and Zeckhauser (1984) and Rosen (1988) in the VoL literature. There was therefore a single model of rational behavior that could be used to discuss both kinds of related issues. However, the additive model was criticized by both literature strands. The HF literature emphasized the limitation of the additive model's intertwining of risk aversion and the intertemporal elasticity of substitution (IES). Some contributions to the VoL literature further found fault with the additive model for implying that agents unavoidably prefer death to life when their consumption gets small enough and when the IES is below one (see e.g., Marshall, 1984 or Rosen, 1988).

Both lines of literature tried to circumvent these difficulties by adopting recursive specifications inspired by the framework of Epstein and Zin (1989) and Weil (1990) – which are usually referred to as EZW preferences. However, the two branches of the literature followed radically different and mutually incompatible choices regarding preference parameters. Due to their focus on saving behavior and portfolio choices, papers in the HF literature typically assume a coefficient of risk aversion above one. This, however, yields EZW specifications that are inadequate for VoL matters, since these specifications imply that people prefer to have shorter lives than longer ones (which we discuss in Section 2.2). To ensure that life is worth living in their models, papers in the VoL literature instead assume a coefficient of risk aversion below one. In this case however, unless the IES is restricted to be greater than one, EZW specifications are ill-defined when applied to realistic mortality patterns and yield counterfactual predictions for lifecycle behaviors (also discussed in Section 2.2).[1]

Considered together, these developments in the HF and VoL literature represent a fragmented approach where a given form of rationality is used when focusing on saving behavior and portfolio choice and another one, incompatible with the former, is used when discussing endogenous changes to mortality risk. These different approaches imply contradictory associations between mortality risk and the propensity to save.[2] This relation between mortality risk and saving behaviors is, however, essential for understanding the impact of ongoing mortality changes on household wealth accumulation both at the micro and the macro levels, and for many issues in the economics of aging. There is therefore a need for clarification of the full role of mortality in models of household behavior, and, above all, for a consistent framework that can be used for jointly modeling choices over savings and choices related to mortality risk while being well-defined when applied to realistic survival probabilities without imposing ad-hoc restrictions on the IES or risk aversion.

The contribution of the current paper is threefold. First, we introduce a flexible, well-behaved recursive framework that can be used in both lines of literature. Central to our approach is that we restrict our attention to models which, like the standard additive expected utility model, are monotone with respect to first-order stochastic dominance.[3] As we explain in the following paragraph, this property is essential for affording an intuitive understanding of how mortality risk and risk aversion together impact household behavior. Like models that use EZW preferences, our specification makes it possible to disentangle the IES and risk aversion, but unlike models with EZW preferences we achieve this in a framework

[1] An exception is the model of Pashchenko and Porapakkarm (2022), who consider non-homothetic EZW preferences, allowing them to combine a positive value of life, an IES below one and a coefficient of risk aversion greater than one. However, this framework is not as tractable as the standard EZW model. See Section 5 for a lengthier discussion.

[2] Recursive models in the HF literature, which usually focus on the case where the IES is below one, find that the propensity to save increases with the likelihood of surviving to the next period. Falls in mortality risk would then generate an increase in (age-specific) savings rates. The opposite association is found in models from the VoL literature, which assume an IES below one.

[3] One may refer to Bommier et al. (2017) for a formal definition of monotonicity with respect to first-order stochastic dominance in intertemporal choice settings.

in which preferences are monotone, and that can further accommodate positive (as well as negative) values of life, independent of the assumptions made regarding the IES or risk aversion. The novelty of this first contribution is that this is the first paper to study lifecycle choices using a framework that disentangles risk aversion from IES while preserving the key characteristics of the standard additive model without imposing constraints on preference parameters (risk aversion, IES, or rate of time preferences).

Second, using this framework, we formally characterize the impact of risk aversion on savings behaviors and annuity demand, in an analytically-tractable two-period framework where mortality is the only risk at play. The results are also shown to extend to a multi-period setting when the value of life is large enough. The innovation of the second contribution is that this paper is the first to derive theoretical predictions explaining how risk aversion simultaneously impacts savings and annuity purchases. These results emphasize the role of the value of life which is shown to be fundamental for understanding the impact of risk aversion on household consumption and saving behavior in the presence of mortality risk.

Last, we provide a quantitative application of our framework to a multi-period and multi-risk setting. We compare the predictions of our model to those of the standard additive model and contrast our results with those of previous studies that used recursive models. The starkest difference between our quantitative model and the standard additive model is in the ability of our model to rationalize low annuity demand. Low annuity demand occurs as a natural outcome of a standard calibration in our model, whereas annuity demand is counterfactually high in the standard model (a manifestation of the so-called 'annuity puzzle' – e.g., Yaari, 1965 or Davidoff et al., 2005). We view this as a channel which can contribute to the literature on determinedly low annuity demand, complementary to other proposed solutions to the 'annuity puzzle' (see Brown, 2007 for a review of this literature). We also find that the role of risk aversion differs from the one found in other studies based on recursive models, precisely because we use a well-behaved specification that features a positive value of life. The novelty of this last contribution is that our quantitative exercise is, to the best of our knowledge, the first attempt to have a flexible life-cycle model that features a wide set of choices (savings, annuity demand, portfolio choice, and bequests), while simultaneously including risks related to survival, labor income and asset return, and imposing a plausible value of life calibration. While a number of these aspects have been considered with standard additive models (see Gomes et al., 2021 for a survey) – which do not allow the analysis of the role of risk aversion in isolation – we are not aware of any contribution where this is done with flexible well-behaved recursive preferences.

The keystone of our contribution is to use a model that fulfills a property of monotonicity with respect to first-order stochastic dominance. Such a property rules out the choice of dominated strategies. Monotonicity has long been seen as a natural aspect of rationality (see for example Arrow, 1951), and it was only in the late 1980s that some non-monotone models, such as EZW preferences, became widely used in applications.[4] The main purpose of EZW specifications was to allow for tractable and flexible models, which were guaranteed by preference homotheticity and the possibility of disentangling risk aversion and IES. This has facilitated the investigation of numerous research questions and has made them an important tool for the study of household behavior. The cost of such tractability and flexibility, though, is the departure from monotonicity, which is not innocuous. An agent endowed with EZW preferences can opt for some choices that offer worse outcomes in all states of the world than other available choices (see Lemma 3 in Bommier et al., 2017). This is analogous to following dominated strategies in a game theoretic context. Moreover, in non-monotone set-ups, risk aversion no longer has a straightforward effect on agents' decisions.

To illustrate this last point, let us take a small detour, and consider an agent who faces the risk of flooding, and can purchase some flood insurance. From an ex-post point of view, if there is flooding, welfare would be improved by having purchased insurance; if there is no flooding, welfare would be hurt by having purchased it. The agent's decision about the quantity of insurance to be purchased has, however, to be taken ex-ante, before knowing whether flooding will occur. Under preference monotonicity, the decision can be seen as involving a trade-off between the welfare obtained in case of flooding and the welfare in case of no flooding. This interpretation of trading-off the welfare in different states of the world is impossible if non-monotone preferences are assumed, since the agent could take a decision that would lower the welfare in all states of the world. In a monotone framework, where choice under uncertainty amounts to trading-off welfare levels obtained in different states of the world, risk aversion simply drives how much weight is given to bad states as compared to good states. In mathematical representations, risk aversion is obtained by affording greater marginal utilities to bad states, for example through the concavity of the utility function (as in the expected utility framework) or through probability transformations (as in the dual model of Yaari, 1987). In our example, if floods are adverse events for the agents, then risk aversion would lead an agent to increase welfare in states of the world where flooding occurs, that

[4]Another well-known model that is non-monotone is prospect theory, in its original formulation (Kahneman and Tversky, 1979). To remedy this non-monotonicity, Tversky and Kahneman (1986) proposed cumulative prospect theory, which is now the most commonly used version.

is, risk aversion would lead them to increase insurance purchase.[5] Note however, that in our example, an inverse relationship would hold if agent saw flooding as a positive event (for example, if flooding improved the fertility of her land).[6] Under preference monotonicity, the relationship between risk aversion and behavior is therefore very intuitive, and naturally depends on whether the occurrence of the uncertain event is perceived as a favorable or as an unfavorable outcome.

Returning to the focus of the current paper, let us now consider a decision problem faced by an agent where the risk is death rather than flooding. An agent who cannot anticipate when she will die, and whose preferences are monotone, has to make trade-offs between the (lifetime) welfare she would obtain if dying young and that obtained if dying old. To improve welfare in the case of a short life, the agent can consume a lot when young (and thus save little) and keep resources to be bequeathed in case of death (and thus invest little in annuities). Such strategies tend to decrease welfare in case of long lives (since consumption will then be low at old ages). The effect of risk aversion on saving and annuity demand depends on whether an early death is seen as an adverse event or not, that is on the sign of the value of life. Under the plausible assumption that the value of life is positive, that is when having a short life is seen as an adverse realization, one should expect risk aversion to decrease savings and annuity purchases – in order to improve the welfare in the bad state of the world. Such intuitive reasoning is formalized in the theoretical part of the paper (Section 3), and is what drives the findings of our quantitative analysis (Section 4). This also explains why our findings differ from those in the HF literature using EZW preferences (Section 5).

Imposing preference monotonicity restricts, of course, the set of admissible preference specifications. While this rules out EZW specifications (except those where the IES is equal to one), it allows for additive and risk-sensitive preferences. The latter preferences, initially introduced by Hansen and Sargent (1995), were shown by Bommier et al. (2017) to be the only recursive preferences that afford the flexibility to separate risk aversion and intertemporal substitutability while preserving monotonicity, recursivity, and the structure of Kreps and Porteus (1978) preferences. Their recursive structure makes them amenable to computational implementation.[7] Risk-sensitive preferences can be seen as a minimal extension

[5]In textbooks, this well-known relation between risk aversion and insurance demand is usually derived in the expected utility framework, but the relation extends to any model which is monotone with respect to stochastic dominance (see e.g., Bommier et al., 2012).

[6]If flooding is a positive event, the agent wants to "sell-short" flood insurance contracts, in other words to hold a negative amount of flooding insurance contracts. The optimum amount to be held then diminishes (i.e., becomes more negative) with agent's risk aversion.

[7]Such preferences are used in applied settings, for instance, in Anderson (2005) or Bäuerle and Jaśkiewicz (2018).

of additive preferences that preserve monotonicity and recursivity but disentangle IES and risk aversion.[8] One of the main benefits of risk-sensitive preferences is that, relying on preference monotonicity, they yield an intuitive interpretation of the impact of risk aversion on household decisions in the presence of mortality risk.

This paper proceeds as follows. In Section 2, we provide a theoretical review of recursive preferences, including additive, EZW and risk-sensitive preferences. In Section 3, we derive theoretical results regarding the impact of risk aversion on saving and annuity demand when the only risk is mortality risk. In Section 4, we outline our quantitative model with multiple periods and multiple risks and discuss its implications for saving behavior and, in particular, annuity demand. In Section 5, we relate our results on the role of risk aversion to previous findings in the HF and VoL literature. Section 6 concludes.

2 Recursive models

2.1 The additive model

The most popular framework in both the HF and the VoL literature is the time-additive expected utility model. To link with the rest of the paper, we provide its recursive definition:

$$U_t = (1-\beta)u(c_t) + \beta E_t[U_{t+1}], \tag{1}$$

where U_t is utility at time t, $\beta \in (0,1)$ is a time preference parameter and $u(c_t)$ is the instantaneous utility derived from consumption at time t.[9] Accounting for mortality is achieved by assigning a utility level, u_d, obtained when death occurs. We follow the VoL literature and assume that this level of utility, u_d, is independent of the age of death (Shepard and Zeckhauser, 1984; Rosen, 1988, among many others).

As it is generally assumed that mortality risk is independent of other risks, the expectation can be decomposed in two stages, one that accounts for the risk of mortality and one that accounts for other risks. Formally denoting the utility conditional on being alive at time t by V_t and the probability of surviving from period t to $t+1$ by π_t the recursion (1) yields:

$$V_t = (1-\beta)u(c_t) + \beta\left(\pi_t E_t[V_{t+1}] + (1-\pi_t)u_d\right). \tag{2}$$

[8] It would be possible to consider other preferences by relaxing recursivity. Recursivity mostly serves tractability and allows us to use standard resolution techniques, such as dynamic programming.

[9] The additive model is frequently defined using the recursion $U_t = u(c_t) + \beta E_t[U_{t+1}]$, which is, of course, equivalent to (1), up to a multiplicative renormalization of the function u.

Additive preferences, as defined in the above equation, are invariant when changing u and u_d (and V_t) by the same positive affine transformation. Therefore, since using $u_d = -\infty$ or $u_d = +\infty$ is not an option (that would lead to a degenerate case where V_t is always equal to $\pm\infty$), it is possible to assume, without loss of generality, that the utility of death is set to zero ($u_d = 0$).[10] If one assumes a constant IES, the function u has to be of the form $u(c) = u_l + K\frac{c^{1-\sigma}}{1-\sigma}$ for some constant u_l (the subscript l stands for "life") and a positive scalar K. The IES is then given by $\frac{1}{\sigma} \neq 1$ – the case $\sigma = 1$ can be derived by continuity. By (multiplicative) normalization, the scalar K can be set equal to 1. The parameter u_l, which determines the utility gap between life and death, is thus an important preference parameter. Setting u_l to zero would not be a mere normalization but would imply making a non-trivial assumption on the value of life. It is noteworthy that when mortality is exogenous, the constant u_l only contributes to an exogenous additive term that has no impact on the ordering of consumption profiles. This explains why the constant u_l is generally ignored in all studies that assume an exogenous mortality pattern but plays an explicit role in the VoL literature (see, for example, the discussion in Hall and Jones, 2007).

The additive specification has been criticized for its lack of flexibility. In line with Epstein and Zin (1989) and Weil (1990), papers in the literature expressed concerns regarding its inability to disentangle the IES from risk aversion.[11]

2.2 Recursive models

The search for greater flexibility led researchers contributing to each of the HF and the VoL literatures to adopt non-additive recursive models for preferences. These models assume that utility U_t at any date t is defined by the following recursion:

$$U_t = f^{-1}((1-\beta)u(c_t) + \beta\phi^{-1}(E_t[\phi f(U_{t+1})]), \tag{3}$$

where ϕ is an increasing function representing risk preferences and f is a normalization device that can be any increasing function. As in the additive model of Section 2.1, $\beta \in (0,1)$ is a time preference parameter and $u(c_t)$ is the instantaneous utility function. The function f has no impact on preferences and its role is to

[10]Indeed, one can define $\tilde{V}_t = V_t - u_d$ and $\tilde{u}(c) = u(c) - u_d$, such that $\tilde{V}_t = (1-\beta)\tilde{u}(c_t) + \beta\pi_t E_t[\tilde{V}_{t+1}]$ and which represents the same preferences as V_t.

[11]In addition, some papers in the VoL literature emphasized the fact that when the IES is smaller than one (i.e., $\frac{1}{\sigma} < 1$), life becomes unavoidably worse than death when consumption gets small enough (see, e.g., Rosen, 1988). This creates an incentive to enter into Russian-roulette games, so as to avoid having to live in a state which is worse than death. This feature may however be seen as a theoretical curiosity, with no consequences in applications as long as consumption stays above the threshold that would make life just as good as death.

facilitate convenient representations of the recursion. A common choice is $f(x) = x$, as in the additive specification (1), and which we will use for the representation of risk-sensitive preferences in Section 2.3. Another common choice for f, most often introduced in the case of EZW preferences involves using the same CRRA function as that used for the instantaneous utility: $f(x) = u(x) = \frac{x^{1-\sigma}}{1-\sigma}$.

As in the additive case, we can derive, from equation (3), the recursion defining the utility conditional on being alive at time t:

$$V_t = f^{-1}\left((1-\beta)u(c_t) + \beta\phi^{-1}\left(\pi_t E_t[\phi f(V_{t+1})] + (1-\pi_t)\phi f(u_d)\right)\right). \tag{4}$$

Here, again, u_d denotes the constant utility level assigned to death and mortality risk is assumed to be independent of other risks.

With this specification, flexibility is gained through the function ϕ, which can be used to vary risk aversion. In particular the greater the concavity of ϕ the greater risk aversion.[12] This gain of flexibility may, however, come with several hurdles. Firstly, if the function ϕ is not of the constant absolute risk aversion kind (i.e., linear or exponential – up to an affine transformation), then the preferences are no longer monotone with respect to first order stochastic dominance (see Bommier et al., 2017 for a longer discussion). Secondly, in search for tractability, most of the literature focused on the homothetic case where the functions u and ϕ are power functions and the constant u_d is set such that $\phi f(u_d) = 0$, yielding the so-called EZW preferences. Unfortunately, in the presence of mortality risks, this fails to provide a decision model that can be used in both the HF and the VoL literature, unless the IES and the coefficient of risk aversion are constrained in very specific ways. An extensive discussion of this aspect can be found in the Online Appendix or in greater detail in Bommier et al. (2021). The main points may however be summarized as follows. With $u(c) = f(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $\phi(x) = \frac{1}{1-\gamma}\left((1-\sigma)x\right)^{\frac{1-\gamma}{1-\sigma}}$ and $f(u_d) = 0$ ($0 \leq \sigma \neq 1$ and $0 \leq \gamma \neq 1$), which correspond to the usual homothetic EZW formulation, equation (4) becomes:

$$V_t = \left((1-\beta)c_t^{1-\sigma} + \beta\pi_t^{\frac{1-\sigma}{1-\gamma}}\left(E_t[V_{t+1}^{1-\gamma}]\right)^{\frac{1-\sigma}{1-\gamma}}\right)^{\frac{1}{1-\sigma}}. \tag{5}$$

Let us further assume that mortality is the only source of uncertainty, so that the amounts consumed in future periods, in case of survival, are known with certainty. The expectation symbol in recursion (5) can then be omitted and an explicit

[12] By Jensen inequality, the concavity of ϕ decreases the certainty equivalent $\phi^{-1}(E[\phi f(U_{t+1})]$ which appears in equation (3).

solution computed:

$$V_t = \left((1-\beta) \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\prod_{s=t+1}^{\tau} \pi_s \right)^{\frac{1-\sigma}{1-\gamma}} c_\tau^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{6}$$

Depending on the values of γ and σ, this solution may have unappealing and strongly counterfacual features. Firstly, when $\frac{1-\sigma}{1-\gamma} < 0$ (i.e. when $\sigma < 1 < \gamma$ or $\gamma < 1 < \sigma$), the series in (6) may be divergent yielding $V_t = 0$, independently of consumption choices. Such a convergence issue occurs as soon as survival probabilities gets small at very old age, which is the case for actual demographic data.[13] Moreover, ignoring such convergence issues, the specification (6) implies a discount rate equal to $\beta^{-1}\pi_t^{-\frac{1-\sigma}{1-\gamma}} - 1$, which increases with age and even becomes negative at old ages (assuming that survival probabilities become small at old ages). In other words, agents become extremely patient with age, which translates into counterfactual consumption profiles in which consumption remains extremely low during most of the life-cycle and skyrockets at old ages. Secondly, when $\gamma > 1$, then $\frac{\partial V_t}{\partial \pi_t} < 0$ implying a negative willingness-to-pay for mortality risk reduction. Agents would be willing to pay to shorten their lives, which contradicts the vast empirical evidence on the positivity of the value of mortality reduction (Viscusi, 2018). Considering both points together, it follows that the only cases where the utility function (6) does not yield strongly counterfactual predictions are those where both γ and σ are smaller than 1 which corresponds to cases where the IES, $\frac{1}{\sigma}$, is larger than 1 and the risk aversion coefficient, γ, less than 1.[14]

Restricting the analysis to cases where the IES is above 1 and risk aversion below 1 is not very satisfactory. Indeed, empirical evidence suggests the IES to be greater than 1 while a low risk aversion cannot be reconciled with asset pricing evidence. Therefore, several papers have considered the EZW specification with σ or γ larger than 1. Most papers in the HF literature actually assume values above 1 for both σ and γ. The fraction $\frac{1-\sigma}{1-\gamma}$ is then positive, granting a positive discount rate, and plausible consumption profiles. The value of life is however

[13]Convergence requires $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} < 1$ for all sufficiently large t. Since $\frac{1-\sigma}{1-\gamma} < 0$, this equivalently puts a lower bound on π_t at old age, and therefore a lower bound on life expectancy.

[14]The limit cases $\gamma = 1$ and $\sigma = 1$ are discussed in the Online Appendix. In brief, when $\gamma \to 1$, the limit of the homothetic model is either degenerate (equal to 0 or infinity regardless of consumption) or exhibits implausible impatience patterns. When $\gamma \neq 1$ and $\sigma \to 1$, it converges to preferences represented by a utility defined by the recursion $V_t = (1-\beta)\log(c_t) + \frac{\beta}{1-\gamma}\log(E_t[e^{(1-\gamma)V_{t+1}}]) + \frac{\beta}{1-\gamma}\log(\pi_t)$, which requires $\pi_t > 0$ at all dates to be well-defined. These preferences satisfy an unappealing separability property, which implies that the marginal rate of substitution between consumption in two different periods is independent of survival probabilities, contradicting the intuition that what makes consumption in a future period valuable is the possibility of being alive in that period.

negative – which as we discuss in Section 3 is not at all innocuous even if mortality is assumed to be exogenous. Papers in the VoL literature focused on the case where $\gamma < 1$ with, as possible "fixes" to accommodate the case $\sigma > 1$, either an assumption of perpetual youth (so that π_t never gets small) or the introduction of a path of exogenous time-varying time preference parameters (i.e., a path of $(\beta_t)_{t\geq 0}$) to compensate for the patience generated by mortality in (6).

Instead of working with models that are non-monotone and suffer from serious limitations (such as systematically predicting a negative value of life), we decided to focus our contribution on monotone preferences. As we will see, this makes it possible to provide new insights on savings, portfolio choices, annuity decisions, and the impact of risk aversion on these decisions.

2.3 Risk-sensitive preferences

As is shown by Bommier et al. (2017), preferences represented by (4) are monotone if and only if the the function ϕ is linear or exponential (up to an affine transformation). This correspond to the so-called risk-sensitive preferences, initially introduced by Hansen and Sargent (1995), where:

$$U_t = (1-\beta)u(c_t) - \frac{\beta}{k}\log\left(E_t[e^{-kU_{t+1}}]\right). \tag{7}$$

Monotonicity arises from the fact that recursion (7) can also be written as $U_t = -\frac{\beta}{k}\log\left(E[e^{-\frac{k}{\beta}((1-\beta)u(c_t)+\beta U_{t+1})}]\right)$, implying that the choices at time t involve maximizing an expectation, just like in the expected utility framework. Interestingly, the IES is not constrained to be constant, since the function u can be arbitrarily general. The parameter k governs risk aversion, where larger values of k are associated with more risk averse behavior. When $k \to 0$, recursion (7) converges toward the standard additive model of Section 2.1. Risk-sensitive preferences with $k > 0$ exhibit both correlation aversion and a preference for early resolution of uncertainty (Stanca, 2023). The intensity of the preference for the early resolution of uncertainty vanishes when β gets close to one (Bommier et al., 2017). The limit case $\beta = 1$ corresponds to the multiplicative model of Bommier (2013) and Bommier and LeGrand (2014) that rules out pure time preferences and preferences for the timing of resolution of uncertainty.[15]

In the context of uncertain lifetime, the recursion (4), giving utility conditional

[15] To be exact, the multiplicative model is obtained by first re-normalizing the instantaneous utility function, setting $\tilde{u}(c_t) = (1-\beta)u(c_t)$ and then taking the limit $\beta \to 1$ while keeping the function $\tilde{u}$ unchanged.

on being alive can be written as:

$$V_t = (1-\beta)u(c_t) - \frac{\beta}{k}\log\left(\pi_t E_t[e^{-kV_{t+1}}] + (1-\pi_t)e^{-ku_d}\right). \tag{8}$$

A feature that is worth emphasizing is that specification (8) is invariant when adding a constant to the instantaneous utility function u. The risk-sensitive model thus preserves one of the invariance properties of the additive specification.[16] This is convenient for normalization matters since it can be assumed that $u_d \in \{0, -\infty, +\infty\}$ with no generality loss.[17] For the remainder of the paper, in studying the role of risk aversion on saving and annuity demand, we use these risk-sensitive preferences. We moreover focus on the cases where $k \geq 0$, so that preferences exhibit correlation aversion, in line with experimental evidence (Andersen et al., 2018).

3 Risk aversion, the value of life and saving behavior

In this section, we provide theoretical predictions regarding the impact of risk aversion on saving behavior and annuity purchase. First, in Section 3.1, we consider a two-period framework and derive general results on the role of risk aversion, emphasizing the importance of assumptions made regarding the sign to the value of life. Then in Section 3.2, we look at the multi-period case, but focus on the case where the value of life is very large.

3.1 A two-period framework

We consider here an agent who lives for at most two periods. In period 0, the agent is endowed with a level of wealth w_0 and must make saving and annuity purchase decisions. More precisely, the agent may invest in bonds, which yield a safe return of R^f and which are bequeathed in case of death, or in annuities, with return $\frac{R^f}{\pi_0}$, but which are not bequeathable. Denoting by c_0 and c_1 the consumption in the first and second periods, by b the amount invested in bonds, by a the amount invested in annuities, and by x the amount bequeathed in case

[16] We provide a detailed proof of this invariance in the Online Appendix. The proof considers bequest motive and extends the invariance result to the normalization of leaving zero bequest.

[17] Specification (8) is in general non-homothetic. An exception corresponds to a constant IES and $k = 0$, providing the standard additive model. Formally, it is also possible to get homotheticity by setting u_d such that $ku_d = +\infty$ and assuming a constant IES. The recursion can then be written as: $V_t = (1-\beta)u(c_t) - \frac{\beta}{k}\log\left(E_t[e^{-kV_{t+1}}]\right) - \frac{\beta}{k}\log(\pi_t)$. When u is log, this falls back to the EZW specification with $\sigma = 1$ discussed in Footnote 14.

of death, we have: $w_0 = c_0 + b + a$, $c_1 = R^f(b + \frac{a}{\pi_0})$, and $x = R^f b$. The agent is endowed with risk-sensitive preferences, as represented by equation (8). However, the utility associated with death is not constant and equal to u_d anymore, but rather depends on the size of the bequest left by the agent. We denote by $v(x)$ the utility associated with bequeathing the amount x. The recursion (8) defining the utility V_0 representing the agent's preferences becomes, in the presence of bequests:

$$V_0 = (1-\beta)u(c_0) - \frac{\beta}{k}\log\left(\pi_0 e^{-k(1-\beta)u(c_1)} + (1-\pi_0)e^{-k(1-\beta)v(x)}\right). \tag{9}$$

We further assume here that $u(c) = u_l + \frac{c^{1-\sigma}}{1-\sigma}$ and $v(x) = u_d + \theta\frac{x^{1-\sigma}}{1-\sigma}$, with $u_d \in \mathbb{R}$ and $\theta \geq 0$. The scalar θ quantifies the strength of the bequest motives. There is no consensus on the formulation of v (known as a 'warm-glow' bequest function), but the form we have chosen is the same as in Cocco et al. (2005), Inkmann et al. (2011) and Yogo (2016).[18] With no loss of generality, we can normalize the utility with $u_d = 0$. Note that further constraining utility by setting, for instance, $u_l = 0$ would not be a mere normalization and would impose constraints on the value of mortality risk reduction. The larger is u_l, the larger is the utility gap between life and death and the larger is the value of mortality risk reduction. In particular, it follows from (9) that:

$$\frac{\partial V_0}{\partial \pi_0} = \frac{\beta}{\pi_0 e^{-k(1-\beta)u(c_1)} + (1-\pi_0)e^{-k(1-\beta)v(x)}} \frac{e^{-k(1-\beta)(u_l + \frac{c_1^{1-\sigma}}{1-\sigma})} - e^{-k(1-\beta)\theta\frac{x^{1-\sigma}}{1-\sigma}}}{-k}, \tag{10}$$

implying that, for given c_1 and w, people prefer longer lives ($\frac{\partial V_0}{\partial \pi_0} > 0$) if u_l is above $\frac{\theta x^{1-\sigma} - c_1^{1-\sigma}}{1-\sigma}$ while people prefer shorter lives ($\frac{\partial V_0}{\partial \pi_0} < 0$) if u_l is below that threshold.

Let b_k and a_k be the optimal saving and annuity choices of an agent with risk aversion k. Formally, the consumption-saving program can be written as:

$$\begin{aligned}(b_k, a_k) = \arg\max_{(b,a)\in\mathbb{R}_+^2} & (1-\beta)u(w_0 - b - a) \\ & - \frac{\beta}{k}\log\left(\pi_0 e^{-k(1-\beta)u(R^f b + R^f\frac{a}{\pi_0})} + (1-\pi_0)e^{-k(1-\beta)v(R^f b)}\right).\end{aligned} \tag{11}$$

We denote by $c_{0,k} = w_0 - b_k - a_k$ and $c_{1,k} = R^f(b_k + \frac{a_k}{\pi_0})$ the corresponding optimal first- and second-period consumption levels. Here again, the case of additively separable preferences is obtained by taking the limit as $k \to 0$.

Proposition 1 *Consider the consumption-saving problem in equation (11).*

[18]Other papers, such as De Nardi (2004), Lockwood (2012) or Bommier and LeGrand (2014) consider a bequest utility of the form $\theta\frac{(\overline{x}+x)^{1-\sigma}}{1-\sigma}$ that is not homothetic but enables bequests to be modeled as a luxury good. We will use such a specification in the calibrated multi-period quantitative model presented in Section 4.

If $k = 0$, then the choices a_0 and b_0, and hence the consumption levels $c_{0,0}$ and $c_{1,0}$, are independent of u_l.

If $k > 0$ and $a_k > 0$ at the optimum, we have:

- *if u_l is such that $\frac{\partial V_0}{\partial \pi_0} > 0$ at the optimum (i.e., if the value of mortality risk reduction is positive), then $\frac{\partial a_k}{\partial k} < 0$, $\frac{\partial b_k}{\partial k} > 0$, $\frac{\partial c_{0,k}}{\partial k} > 0$ and $\frac{\partial c_{1,k}}{\partial k} < 0$;*
- *if u_l is such that $\frac{\partial V_0}{\partial \pi_0} < 0$ at the optimum (i.e., if the value of mortality risk reduction is negative), then $\frac{\partial a_k}{\partial k} > 0$, $\frac{\partial b_k}{\partial k} < 0$, $\frac{\partial c_{0,k}}{\partial k} < 0$ and $\frac{\partial c_{1,k}}{\partial k} > 0$.*

The proof can be found in Appendix A. To the best of our knowledge, Proposition 1 is the first result that characterizes the role of risk aversion simultaneously for riskless savings and annuity investment and that emphasizes the key role of the sign of the value of life in these relationships.

Here we will comment on the results of Proposition 1, focusing on the case where $k > 0$ and $a_k > 0$ (which avoids corner solutions). We begin with the case where $\frac{\partial V_0}{\partial \pi_0} > 0$ at the optimum, that is, when the agent would prefer to have a greater survival probability. We find that increasing risk aversion decreases second-period consumption ($\frac{\partial c_{1,k}}{\partial k} < 0$), implying therefore that overall savings payoffs are reduced ($\frac{\partial}{\partial k}(b_k + \frac{a_k}{\pi_0}) < 0$). Moreover, savings shift towards safe assets ($\frac{\partial b_k}{\partial k} > 0$) at the expense of annuity purchases ($\frac{\partial a_k}{\partial k} < 0$). Intuitively, as discussed in the introduction, with monotone preferences, a utility maximizing choice under uncertainty can be seen as involving trade-offs between the ex-post utilities obtained in all states of the world. Furthermore, increasing risk aversion can be understood as putting greater weight on "bad states" of the world. Here, there are two states (death or survival at the end of the first period) and the inequality $\frac{\partial V_0}{\partial \pi_0} > 0$ indicates that the agent considers as the bad state the case where she dies after the first period. To increase lifetime utility derived in the case that the bad state is realized, she increases her first period consumption ($\frac{\partial c_{0,k}}{\partial k} > 0$) and the amount she leaves as bequest ($\frac{\partial b_k}{\partial k} > 0$). At the same time, she purchases a smaller amount of annuities. These choices make a short life a less adverse outcome. Of course, this comes at the cost of a lower second-period consumption ($\frac{\partial c_{1,k}}{\partial k} < 0$) and of a lower lifetime utility in case of survival (we prove in Appendix A that $\frac{\partial}{\partial k}(u(c_{0,k}) + \beta u(c_{1,k})) < 0$).

Interestingly, the findings look very different in the (counterfactual) case where $\frac{\partial V_0}{\partial \pi_0} < 0$, that is, when the agent would prefer to have a lower survival probability. As shown in Proposition 1, the impact of an increase in risk aversion is systematically reversed compared to the case where the value of mortality risk reduction is positive. This follows intuitively from the fact that the impact of an increase in risk aversion is to put greater weight on the bad state, and thus mechanically depends on which state is actually the worst. When $\frac{\partial V_0}{\partial \pi_0} < 0$, the bad state is the one where the agent lives for two periods, and an agent with $k > 0$ purchases a larger amount of annuities than an agent with additive preferences ($k = 0$), precisely to make such a "bad outcome" not so bad. As we will see in Section 5, such matters related to the sign of the value of life are key to understanding how our results contrast with those of previous contributions.

3.2 The general case with an infinite value of life

An interesting aspect of the life-cycle model with risk-sensitive preferences, is that, when mortality is exogenous, the model admits a well-defined limit when the value of life becomes infinitely large. Moreover, this limit model has a simple additive structure that makes it possible to derive formal results. Formally consider the multi-period version of equation (9) assuming a general (increasing and concave) bequest motive function normalized so that $v(0) = 0$:[19]

$$V_t = (1-\beta)u(c_t) - \frac{\beta}{k}\log\left(\pi_t e^{-kV_{t+1}} + (1-\pi_t)e^{-k(1-\beta)v(x_{t+1})}\right). \tag{12}$$

where V_t and c_t are the utility and consumption in period t conditional on being alive, and x_{t+1} is the amount left in bequest if dying at the end of period t and π_t the probability of surviving from period t to period $t+1$. Assume that $u(c) = u_l + \frac{c^{1-\sigma}}{1-\sigma}$ and consider the limit where u_l tends to infinity while maintaining the product ku_l equal to a constant, which we denote by κ below. This involves taking the limit where value of life gets increasingly large while having a risk aversion with respect to life duration that converges towards a finite limit.[20] It can be shown through a first-order Taylor expansion that the risk-sensitive utility, V_t, can then be approximated by $V_t \simeq \frac{1}{k}\log(z_t) + V_t^\infty$, where V_t^∞ and z_t are recursively defined by:

$$V_t^\infty = (1-\beta)\frac{c_t^{1-\sigma}}{1-\sigma} \tag{13}$$
$$+ \frac{\beta}{\pi_t + (1-\pi_t)z_{t+1}}\left(\pi_t E_t[V_{t+1}^\infty] + (1-\pi_t)z_{t+1}E_t[(1-\beta)v(x_{t+1})]\right),$$

$$\log(z_t) = (1-\beta)\kappa - \beta\log\left(\pi_t/z_{t+1} + 1 - \pi_t\right). \tag{14}$$

[19]With such a normalization the continuation utility when dead, $(1-\beta)v(x_{t+1}) + \beta v(0)$, simplifies to $(1-\beta)v(x_{t+1})$.

[20]In the limit $u_l \to \infty$ with $ku_l = \kappa$, we can show that there exits a probability $\tilde{\pi} = \frac{1-e^{-\kappa(1-\beta)}}{1-e^{-\kappa(1-\beta)(1+\beta)}} > \frac{1}{2}$ such that the agent is indifferent between (i) living one extra period for sure (deterministic life duration of one period), or (ii) living either two extra periods with probability $\tilde{\pi}$ or zero periods with probability $1-\tilde{\pi}$ (uncertain life duration, with a life expectancy of $2\tilde{\pi} > 1$). The larger κ, the larger $\tilde{\pi}$, explaining why κ can be interpreted in terms of risk aversion with respect to life duration.

Under exogenous mortality, $\log(z_t)$ is exogenous, and maximizing V_t is thus equivalent to maximizing V_t^{∞}. For a large value of life, the risk-sensitive model therefore converges to the model represented by V_t^{∞}. The recursive equation that defines V_t^{∞} is similar to the usual additive specification but embeds an (exogenous) age-dependent discount factor $\frac{\beta}{\pi_t+(1-\pi_t)z_{t+1}}$ and an age-dependent weight z_{t+1} that applies to bequest utility term.

Similar to the two-period problem discussed in Section 3, consider the case of an agent endowed with wealth w_t in period t who has to chose her bond savings, $b_{t,\kappa}$, and annuity purchase, $a_{t,\kappa}$. These amounts determine how much the agent consumes in period t (i.e., $c_{t,k} = w_t - b_{t,\kappa} - a_{t,\kappa}$), how much wealth she would get in period $t+1$ if remaining alive ($w_{t+1} = R^f(b_{t,\kappa} + \frac{a_{t,\kappa}}{\pi_t})$), and the bequeathed amount in case she dies at the end of period t (i.e., $x_{t+1} = R^f b_{t,\kappa}$). When the agents choose these quantities to maximize V_t^{∞}, then the following result holds.

Proposition 2 *Consider the limit case of an infinite value of life introduced above and assume an interior solution with positive annuity purchase. Then, for a given amount of wealth held at age t, the savings, $b_{t,\kappa}$, and annuity purchases, $a_{t,\kappa}$, at time t are such that:*[21]

$$\frac{\partial a_{t,\kappa}}{\partial \kappa} < 0, \ \frac{\partial b_{t,\kappa}}{\partial \kappa} > 0 \text{ and } \frac{\partial c_{t,\kappa}}{\partial \kappa} < 0.$$

The proposition 2 shows that when the value of life is very large (and thus positive), most of the results derived in the two-period setting of Section 3.1 extend to the multi-period case. The marginal propensity to consume increases with risk aversion ($\frac{\partial c_{t,\kappa}}{\partial \kappa} = -\frac{\partial (b_{t,\kappa}+a_{t,\kappa})}{\partial \kappa} > 0$). This follows from a reduction in annuity purchase ($\frac{\partial a_{t,\kappa}}{\partial \kappa} < 0$) which more than compensates the increase in savings ($\frac{\partial b_{t,\kappa}}{\partial \kappa} > 0$). However, unlike in the two-period case of Proposition 1, the impact of risk aversion on the next period consumption is ambiguous: the sign of $\frac{\partial c_{t+1,\kappa}}{\partial \kappa}$ is unclear. This comes from two opposite effects. First, as in the two-period case, date-t savings are lower. Wealth available in period $t+1$ is thus reduced which has a negative impact on consumption at date $t+1$. However, this effect is counter balanced by an impatience effect, implying that the agent wants to increase date-$t+1$ consumption at the expense of future ones. The overall effect is thus ambiguous.

[21] We hold w_t constant as we are interested in how the marginal propensity to save and to purchase annuities varies with κ. Of course from a life-cycle perspective, the wealth held at age t will also depend on κ which will generate wealth effects that will add to those discussed here.

4 A quantitative lifecycle model

Having argued why the value of life matters even in models with exogenous mortality risk, this section outlines a multi-risk, multi-period, quantitative lifecycle model which can be used to study the interplay between that risk, saving behavior, portfolio choice and annuity purchases. The model's innovation is to ensure, by using risk-sensitive preferences, preference recursivity and monotonicity while matching empirical estimates of the value of mortality risk reduction. The model structure is very rich. It features risks that affect: (i) mortality, (ii) income, and (iii) asset returns. Agents also face a complex portfolio choice that involves a riskless asset, a risky asset, and annuity holdings.

4.1 The setup

We consider an economy of agents endowed with risk-sensitive preferences who face risks over mortality, income and asset returns. Agents may save through a bond, a risky asset and may insure against longevity risk by purchasing an annuity. Time is discrete, a model period is a year, and time t corresponds to age minus 18. Agents enter the model at the start of working life, at $t = 0$. There is a single consumption good, whose price serves as a numeraire.

Mortality risk. Agents face mortality risk, which is assumed to be exogenous and independent of all other risks. If alive at date t, agents survive to date $t+1$ with probability π_t. There exists a date T_M, such that the probability of living after T_M is $\pi_{T_M} = 0$.

Labor income risk. At any age, when alive, agents receive an income denoted y_t. They exogenously retire at date T_R. During retirement ($t \geq T_R$), agents receive annual public pension (Social Security) income $y_t = y^R$. During working life ($t < T_R$), agents earn a risky labor income $y_t = y_t^L$, defined by $\ln y_t^L = \mu_t + \zeta_t$. The sequence $(\mu_t)_{t\geq 0}$ is a deterministic process that depends on age (see Appendix B.1 for further details), and $(\zeta_t)_{t\geq 0}$ is an AR(1) stochastic component, with persistence parameter ρ and innovation $(\upsilon_t)_{t\geq 0}$, which is IID normally distributed with mean 0 and variance σ_υ^2. We denote average earnings over working life as $\bar{y}$.

Financial risk and security markets. Agents can save through bonds and stocks and can purchase an annuity. The bond pays a constant risk-free gross return, R^f. The stock yields a risky return, defined as: $R_t^s = R^f + \omega + \nu_t$, where ω represents the average risk premium of stocks over bonds, while the stochastic component of the risky return $(\nu_t)_{t\geq 0}$ is an IID normally distributed process with mean 0 and variance σ_ν^2.

Agents must pay a cost $F \geq 0$ to participate in the stock market, which may be interpreted as the opportunity cost of discovering how the stock market works. We assume it is a flat once-in-a-lifetime cost: if the cost is paid at a given date t by an agent, they can freely trade stocks at date t and at any date afterwards.

Finally, an annuity can be purchased in the period before retirement $(T_R - 1)$. The annuity is a financial asset that pays one unit of income every period from T_R, as long as its holder is alive. The price of a single unit of annuity income, q, is:

$$q = (1+\delta) \sum_{\tau=1}^{T_M - T_R} \frac{\prod_{s=0}^{\tau} \pi_{T_R - 1 + s}}{(R^f)^\tau}, \tag{15}$$

where the parameter $\delta \geq 0$ is a loading factor on annuity. When $\delta = 0$, the annuity is actuarially fair and its price equals the discounted present value of future payoffs. The larger is δ, the farther is annuity pricing from actuarial fairness. There is one final annuity market imperfection. Following Pashchenko (2013), annuity purchases below a minimum threshold ($\underline{a} > 0$) are not allowed.

Choices and constraints. If an agent is alive, her resources at the beginning of the period consist of her wealth, comprising bond, annuity, and stock payoffs plus labor income earned, or public pension income received, in the period. Resources are used for consumption as well as the purchase of bonds, annuities, and stocks. The budget constraint of a living agent at date t can then be expressed as follows:

$$c_t + qa_t + b_t + s_t + 1_{\eta_t=1} 1_{\eta_{t-1}=0} F = y_t + w_t, \tag{16}$$

$$\text{with: } w_t = a_{T_R-1} 1_{t \geq T_R} + R^f b_{t-1} + R_t^s s_{t-1}, \tag{17}$$

where c_t and w_t are consumption and wealth in period t and b_t, s_t and a_t are, respectively, the quantity of bonds, stocks and annuities purchased in period t. The index η_t reflects market participation status and is equal to 0 if she has never paid the participation cost before and therefore never held stocks. The term $1_{\eta_t=1} 1_{\eta_{t-1}=0} F$ in equation (16) represents the fixed once-in-a-lifetime cost of participation. Annuity income is received from age T_R and is therefore equal to $a_{T_R-1} 1_{t \geq T_R}$. No asset, including annuities, can be sold short. These constraints are summarized here:

$$s_t = 0 \text{ if } \eta_t = 0, \tag{18}$$

$$a_t = 0 \text{ if } t \neq T_R - 1, \tag{19}$$

$$a_{T_R-1} = 0 \text{ or } a_{T_R-1} \geq \underline{a}, \tag{20}$$

$$b_t \geq 0, s_t \geq 0 \text{ and } c_t > 0. \tag{21}$$

If an agent is dead at date t, she bequeaths bonds and stocks, but not annuities. The bequest x_t amounts to:

$$x_t = R^f b_{t-1} + R_t^s s_{t-1}. \tag{22}$$

A feasible allocation is a sequence of choices $(c_t, b_t, a_t, s_t, x_t, \eta_t)_{t\geq 0}$ satisfying the constraints (16)–(22).

4.2 Preferences and agents' program

Intertemporal preferences. Agents have risk-sensitive preferences. The utility of an agent at age t when alive, V_t, is defined through the recursion (12), where we still normalize the utility, such that $v(0) = 0$.

Instantaneous utility function specification. We assume that agents have a constant IES. Formally,

$$u(c) = \begin{cases} u_l + \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma \neq 1, \\ u_l + \log(c) & \text{if } \sigma = 1, \end{cases} \tag{23}$$

where $\sigma > 0$ is the inverse of the IES, and u_l is a parameter that provides the instantaneous utility derived when alive and consuming one unit of consumption ($u(1) = u_l$). It can also be interpreted as the difference in utility between being alive and consuming one unit and being dead and bequeathing nothing ($u_l = u(1) - v(0)$). Since utility has already been normalized when assuming $v(0) = 0$, we cannot additionally set u_l to an arbitrary value. This parameter must, therefore, be carefully calibrated. Note that when $\sigma > 1$, there necessarily exists a threshold $\underline{c}$ below which $u(c) < 0$, implying that agents would prefer to die and leave no bequest rather than staying alive. In theory, this threshold could be used to calibrate u_l. However, this would involve basing the calibration on extreme cases (suicides), for which our model is surely ill-suited (a suicidal decision is a complex multi-dimensional decision that, needless to say, involves more than poverty). A preferable calibration strategy, which we will use in Section 4.4, relies on using agents' decisions that relate to safety. Since a higher value of u_l means a higher valuation of being alive relative to being dead, the value of u_l should be reflected in the financial decisions that agents make to lower their mortality risks, for example when investing in safer (but more expensive) cars or opting for safer (but lower paid) jobs.

The utility derived from bequests, $v(x)$, is assumed to be continuous, increasing in the bequest amount, and to exhibit bounded and decreasing marginal utility. The functional form we use has been widely applied (see e.g., De Nardi, 2004, De Nardi et al., 2010, Ameriks et al., 2011, and Lockwood, 2012 and 2018). Formally,

$$v(x) = \begin{cases} \frac{\theta}{1-\sigma}\left[(\overline{x}+x)^{1-\sigma} - \overline{x}^{1-\sigma}\right] & \text{if } \sigma \neq 1, \\ \theta \log\left(\frac{\overline{x}+x}{\overline{x}}\right) & \text{if } \sigma = 1, \end{cases} \tag{24}$$

where σ is the inverse of the IES used in the expression (23) defining the function u, while $\theta \geq 0$ governs the strength of the bequest motive. With $\overline{x} > 0$, bequests are a luxury good, as has been shown by, for example, Hurd and Smith (2002). The derivative $v'(0)$ is finite, so that agents bequeath only when their wealth is large enough (an empirical regularity documented by e.g., De Nardi, 2004).

Agents' program. The agents' problem involves determining the feasible allocation that maximizes utility defined in (3). There is no analytical solution to this problem. We therefore solve the model numerically. In short, state variables are discretized, and decision rules are obtained for points on the grid by backwards induction from the last period. Linear interpolation is used to evaluate the value function at points off the grid and integration over earnings and asset price shocks is carried out using Tauchen (1986). Further details are given in the Online Appendix.

4.3 Value of mortality risk reduction

To properly calibrate u_l, we will consider the marginal rate of substitution between survival probability and wealth, which quantifies how much a given agent is willing to pay – in terms of wealth – to reduce her mortality risk. This marginal rate of substitution is most often called the *value of a statistical life* (VSL, henceforth), though there have been recent recommendations to use the terminology "value of mortality risk reduction".[22] The VSL at date t, denoted VSL_t, is defined as:

$$VSL_t = \frac{\frac{\partial V_t}{\partial \pi_t}}{\frac{\partial V_t}{\partial w_t}}, \tag{25}$$

where w_t is wealth and is given in equation (17). This definition is standard and is used in Rosen (1988), for instance. The formal expressions for the risk-sensitive and additive models – which will be used to calibrate u_l – can be found in the Online Appendix.

Empirical literature on the value of a statistical life. The value of mortality risk reduction is a central parameter for cost-benefit analyses in many policy realms. This includes evaluating environmental policy (see US Environmental Protection Agency 2011 where the value of mortality risk reduction is central in estimating the benefits of the Clean Air Act), transport policy (see US Department of Transportation, 2021 on quantifying the benefit of road safety rules) and health policy (Murphy and Topel, 2006 and Hall and Jones, 2007 or, in the context of the Covid-19 pandemic, Greenstone and Nigam, 2020, Hall et al. 2020, Hammitt, 2020, and Robinson et al., 2021).

There are two distinct approaches that have been used to estimate the value of mortality risk reduction. The first is a revealed preference approach which estimates it from observed decisions by individuals (e.g., from compensating differentials associated with risky jobs or willingness to pay for safety features on vehicle purchases). The second is a stated preference approach, where individuals' valuations are explicitly elicited by a survey. Both approaches provide a relatively broad range of estimates. This is not surprising, as any estimate of willingness-to-pay for mortality risk reduction will depend on individual preferences and individual financial and demographic characteristics. However, there is a consensus that the willingness-to-pay for mortality risk reduction is positive and large. Broad overviews of the literature and details on the range of estimates that have been reported can be found in Viscusi and Aldy (2003) and Kniesner and Viscusi (2019).

4.4 Calibration

Demographics, endowments and asset market parameters. We describe here how externally-set parameters, relating to demographics, endowments, and

[22]See `https://www.epa.gov/environmental-economics/mortality-risk-valuation` for a discussion. A more precise terminology would actually be the "value of *marginal* mortality risk reduction", to emphasize that this marginal rate of substitution should not be directly used to compute the willingness to pay for non-marginal risk reduction. See e.g., Hammitt (2020) or Robinson et al. (2022).

asset markets, are chosen. Table 1 provides a summary. All monetary values are in 2022 US dollars unless specified otherwise.

Table 1: Externally-set parameters.

Parameter	Value	Source
Demographics		
Retirement date, T_R	47 (= 65 − 18)	SSA Historical Normal Retirement Age in US
Maximal life duration, T_M	82 (= 100 − 18)	
Cond. survival rates, $\{\pi_t\}$		US Life Tables for a male cohort born in 1940 (Bell and Miller, 2005)
Endowments		
Average wage, $\bar{y}$	US$ 43,104	Lifecycle average for 1940s men (own estimates)
Age productivity, $\{\mu_t\}$		Lifecycle average for 1940s men (own estimates)
Public pension, y^R	40%×$\bar{y}$	Average SS replacement rate (Biggs and Springstead, 2008)
Labor income autocorr., ρ	0.977	Own estimates
Var. of persistent shocks, σ_v^2	0.010	Own estimates
Asset Markets		
Gross risk-free return, R^f	1.02	Campbell and Viceira (2002)
Equity premium, ω	4%	Campbell and Viceira (2002)
Stock volatility, σ_ν	15.7%	Campbell and Viceira (2002)
Annuity Market		
Min. annuity purchase, $\underline{a}$	US$ 4,480	Pashchenko (2013)
Administrative load, δ	10%	Pashchenko (2013)

Notes: Dollars are 2022 dollars.

Taking demographics first, economic life is modeled as starting at the age of 18 and agents retire at the age of 65, which for many years was the "normal retirement age" for Social Security in the US. Mortality rates are taken from the US Life Tables for men born in 1940 (Bell and Miller, 2005). We assume that all individuals die at the age of 100 if not before.

Turning to endowments, agents earn a wage in each period up to the age of 64 and receive Social Security payments from the age of 65. We estimate the deterministic and stochastic components of the earnings process using data from the Panel Study of Income Dynamics. We focus on the earnings trajectory of a one particular cohort (men born in the 1940s).[23] We discuss the estimation of

[23]The reason for this restriction is that in the cohort that we focus on, those born in the 1940s,

these objects in Appendix B.1. We find an autoregressive parameter $\rho = 0.977$ and a variance of persistent shocks of $\sigma_v^2 = 0.010$. These values are close to those estimated by Guvenen (2009), who estimates an autoregressive parameter $\rho = 0.988$ and a variance of persistent shocks of $\sigma_v^2 = 0.015$. Public pensions, y^R, are set at 40 percent of the average simulated earnings in our modeled economy, which is approximately the average replacement rate afforded by Social Security (Biggs and Springstead, 2008). We assume that agents enter the model with no assets, $s_{-1} = 0$ and $b_{-1} = 0$.

Turning to the features of our asset market, we follow Campbell and Viceira (2002). The gross risk-free return is set at $R^f = 1.02$. We set the equity premium to $\omega = 4\%$ and stock volatility is $\sigma_\nu = 15.7\%$. These values represent common choices in the lifecycle literature (see, for example, Lusardi et al., 2017). We set the administrative load (δ) to 10% (as in Pashchenko, 2013, Lockwood, 2012, and in the range of Brown, 2007). We set the minimum annuity purchase to $4,480, calculated by converting the value in Pashchenko (2013) to 2022 dollars. The final feature of the asset market, F, the once-in-a-lifetime participation cost in the risky asset, is calibrated to match participation in stock markets – we defer discussion of that to below.

Preference parameters. We set the IES to 0.5, so that its inverse is $\sigma = 2$, which is a common value in the literature. The time preference and risk aversion parameters, β and k, and the utility gap between life and death, u_l, are set so that lifecycle behaviors match our calibration targets, which we will detail later.

To set the bequest function parameters (θ, which governs the intensity of the bequest motive, and $\bar{x}$, which governs the extent to which bequests are a luxury good), we use the estimates of De Nardi et al. (2010), who study the problem of bequests in detail. Their model, which features additive preferences, implies values for the IES and the time preference parameter that differ from ours, and we therefore cannot directly use their bequest parameters. Our approach is to replicate two targets implied by their estimates: (i) the maximal wealth at which no bequest is left; and (ii) the marginal propensity to consume wealth, both of which are computed for a living agent at the maximal age – such that she dies for sure in the next period – who can only save in the riskless asset. This removes any risk in the model and the risk-sensitive model reduces to the additive one. The parameters of De Nardi et al. (2010) imply a value of $36,000 (in 1998 dollars, or $64,635 in 2022 dollars) for the maximal no-bequest wealth and of 0.88 for the

labor market trajectories for women were more frequently interrupted than in the present day, and we do not wish these (often planned) transitions in wages to appear as result of the resolution of risk.

marginal propensity to bequeath wealth in the last period of life.[24] To match these values, we set $\bar{x} = 11.2$ and $\theta = 58.8$.[25] See Appendix B.3 for further details on the calculation of these numbers.

Calibration of remaining parameters. Four parameters are calibrated to match features of behavior over the lifecycle. These parameters are the utility gap between life and death (u_l), the time preference parameter (β), the parameter governing risk aversion (k), and the cost of participating in stock markets (F). We calibrate them by matching four targets: the VSL at age 45, asset holdings at age 65, annuity holdings at 65 and the proportion of people who have held risky assets by the age of 65.[26] The target values are summarized in Table 2.

Table 2: Calibration targets.

Target	Value	Source
VSL	\$12.9m	US Department of Health and Human Services (2016)
Median wealth at age 65	\$230,000	Health and Retirement Study
Proportion holding annuities at age 65	5%	Pashchenko (2013)
Stock market participation rate at age 65	65%	Health and Retirement Study

Notes: See the text for calculation and further details on source. Dollars are 2022 dollars.

For the VSL at age 45, we target a mean value of \$12.9m, which corresponds to the central estimate suggested by US Department of Health and Human Services (2016).[27] This is close to the \$12.5m suggested by US Department of Transportation (2023), the value of \$10.7m advised by the Environmental Protection Agency (2023) (\$7.4m in 2006 dollars) and to the \$10m suggested as a central estimate for the US in the recent review article by Kniesner and Viscusi (2019). In Section 4.5, we also show the sensitivity of our results to using the Low VSL estimates (\$6.1m) and High VSL estimates (\$19.7) from US Department of Health and Human Services (2016), where once again all numbers are converted to 2022 prices.

[24]These numbers are implied by the estimates in column 3 of Table 3 of their paper. See the discussion in Appendix D of their paper.

[25]$\bar{x}$ is expressed in units of average income. The dollar equivalent is \$480,000.

[26]For the value of life we choose its estimated value at age 45 (and not 65) as a calibration target, as most empirical studies rely on wage-risk trade-offs and are estimated on samples comprising those of working age.

[27]The central estimate is given in 2014 dollars (\$10.4m). We convert it to 2022 dollars for consistency with all other monetary values. See Viscusi (2021) for a discussion on updating VSL values.

To obtain a target to match median modeled wealth at the age of 65, we use the Health and Retirement Study. We select a sample that is as close as possible in composition to the sample we use to estimate our earnings process. We define a sample of men born in the 1940s whom we observe as unmarried in each wave from wave 4 of the Health and Retirement Study. We pool data across waves and calculate median wealth which is approximately of $230,000. We also show in Appendix C.2 how our simulated profiles match selected untargeted wealth moments.

We target the proportion of households holding annuities. Annuities are not popular (this fact is, of course, the source of the annuity 'puzzle'). In our small sample of not recently-married men born in the 1940s, we observe approximately 3% with annuity income over the age of 66, though given the small sample size, and the fact that annuity purchase is a rare event, we also broaden the analysis to men in all cohorts observed after the age of 65, finding that 7% of men are observed with some annuity income. These figures line up with those found elsewhere: Lockwood (2012) reports that 3.6% of single retirees hold annuities. Pashchenko (2013) reports 5%, also for a sample of single retirees. Reflecting the robust finding that annuity demand in the US is positive but low we target a proportion of 5% for annuity holders.

Finally, the once-in-a-lifetime participation cost is calibrated (jointly with the preference parameters) to match a measure of stock market participation. For the population we study, we target a stock market participation rate of 65% at age 65, which is the proportion of our sample who we observe holding equities in any wave of the Health and Retirement Study up to the wave 10 (in 2012, when our cohort of interest were aged between 62 and 72). See Appendix B.2 for further details. This is close to the peak equity ownership reported by Alan (2006).

In our results, we will compare our simulated profiles to an additive model that will also be calibrated to meet these targets. As the risk aversion parameter k is constrained to equal zero in the additive specification, to ensure we are comparing across models with the same number of free parameters, we add another degree of freedom. To do this, rather than imposing the 10% administrative load (δ), informed by the literature, we find the administrative load that would rationalize annuity demand.

4.5 Results

Parameter estimates. Table 3 gives our baseline parameter estimates and shows, for comparison, the parameter estimates for the additive model. The reported values assume that one unit of consumption corresponds to the average income $\bar{y}$. The estimate for our risk-sensitive model of β, 0.975, is close to values typically estimated in lifecycle models and additionally is very close to the value (0.97) assumed by Pashchenko (2013) and Lockwood (2012), two papers which try to rationalize low annuity-demand. De Nardi et al. (2010), on whose estimates we base our parameterization of the bequest function, also use a value of 0.97.

The specific values we obtain for u_l and k depend on normalization choices we made, and are best understood by thinking of the trade-off between consumption and life duration (for u_l) or by looking at how agents compare different lotteries over life duration (for k). Regarding u_l, consider a setting with an agent whose consumption equals the average annual income. Such an agent endowed with the calibrated value of u_l would be willing to give up about 6% of her consumption during her last year of life in exchange for one extra week of life.

The value of k is set to 1.02. The value of this parameter cannot, of course, be directly compared to coefficients of risk aversion in EZW models: k is only one of the determinants of risk aversion with respect to consumption (defined as $-c_t \frac{\partial^2 V_t}{\partial c_t^2} / \frac{\partial V_t}{\partial c_t}$) that is also driven by other preference parameters, such as σ (as in the additive model). For an interpretation of the magnitude of k, consider the following situation. A 65 year-old agent, endowed with the set of preferences estimated in our baseline model is faced with a previously unanticipated option to undergo some surgery, knowing that the surgery would increase her life expectancy, but would involve taking a 5% risk of an immediate death. With our estimated value of k, an agent would opt for surgery only if the increase in life expectancy (taking account of the risk of dying in the operation) exceeds 10.5 months.[28] Had we set $k = 0$ (and kept all other parameters the same), the agent would opt for the operation when the increase in life expectancy is as short as 2.4 months.

The once-in-a-lifetime participation cost, at 111% of average annual income, is necessarily large to match observed (non-)participation in risky assets when the equity premium is 4%, (see, for example, Mehra and Prescott, 1985 and Kocherlakota, 1996). However, note that this is paid only once per life. To place this quantity in perspective, for those who pay the participation cost, it represents 1.7% of lifetime consumption (discounted by the risk-free rate).[29]

[28]To calculate this, we find the scalar that, when multiplied by all survival probabilities for ages after 65, exactly compensates agents, in terms of (ex-ante) utility, for the loss in utility associated with the additional 5% chance of dying at the age of 65. We can then use this quantity to calculate the new life expectancy taking into account both the risk of dying in the operation and the greater survival in each period if she survives.

[29]This value is consistent with the literature. For instance, Favilukis (2013) sets a cost that is a combination of once-per-lifetime and once-per-period payments. The one-per-life cost is 2.1% of median wealth and the per-period cost 10% of this value. However, this cost must be put in the

Table 3: Estimated parameters in baseline economy

Parameter	Risk-Sensitive	Additive Model
Preferences		
Inverse of IES, σ	2.0†	2.0†
Risk aversion parameter, k	1.02	0.000†
Life-death utility gap, u_l	4.3	15.8
Discount factor, β	0.975	0.953
Bequest motive strength, θ	58.8†	58.8†
Bequest luxury good, $\overline{x}$	11.2†	11.2†
Annuity administrative load, δ	10%†	23%
Asset Markets		
Once-in-a-lifetime participation cost, F	111% of $\bar{y}$	123% of $\bar{y}$

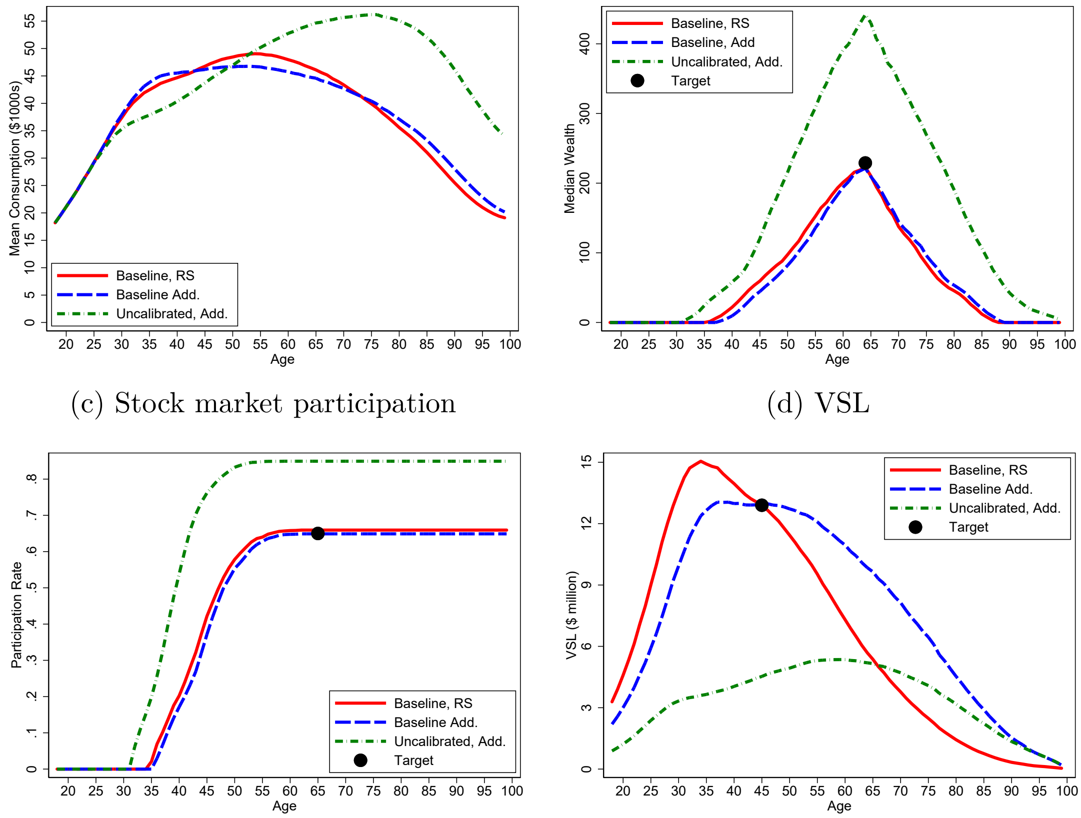
Notes: One unit of consumption is equal to $\bar{y}$. Quantities indicated by a † are imposed rather than estimated.

Estimated profiles. Figure 1 shows profiles of mean consumption, wealth, participation in stock market and VSL over the lifecycle for a simulated sample of individuals. Results are reported for three specifications. The first two correspond to the calibrated versions of the risk-sensitive (referred to as "Baseline RS" or "RS" in short) and additive (referred to as "Baseline Additive") models, whose parameters are shown in Table 3. Both models, by construction, closely match the quantitative targets which are marked on the graphs and predict that the proportion of individuals holding annuities is 5%. The third specification, labeled as "uncalibrated-additive" corresponds to the model obtained while keeping all parameters of the risk-sensitive case fixed, but setting $k = 0$ to recover an additive specification. We show these results to facilitate comparative statics with respect to the risk aversion parameter k.

Comparative risk aversion. RS agents are more risk averse than the "uncalibrated-additive" agents, but are identical in all other aspects. Comparing the predictions of the RS model with those of the "uncalibrated-additive" model therefore reflects the theoretical predictions of Section 3 about the role of risk aversion. The framework here, though, is a richer one than that outlined in Section

context of the low time preference parameter ($\beta = 0.78$) and a low IES ($\sigma = 10$) of that paper's calibration. In their baseline calibration with no catastrophic risk, Fagereng et al. (2017) report a per-period cost of approximately \$ 350 (1995 USD), but the participation rate exceeds 90%, despite a low time preference parameter and a low IES ($\beta = 0.827$ and $\sigma = 14.444$). Introducing catastrophic risk in that paper reduces the participation cost, albeit by diminishing the risk premium to a lower value than that we use in this paper. They further provide the outcomes of simulations with $\beta = 0.96$, $\sigma = 10$, and a per-period participation cost of \$ 300. In that case, the participation rate hits 100% at age 45 – highlighting the crucial role of β.

Figure 1: Lifecycle profiles for the baseline calibration.



(a) Consumption

(b) Median wealth

(c) Stock market participation

(d) VSL

3 – in particular with the addition of income and asset return uncertainty. These additions are likely to have important effects: income uncertainty generates a precautionary savings effect that is amplified with risk aversion.[30] The negative relationship between risk aversion and savings derived in Section 3 will thus be complemented by a precautionary effect that would imply an opposed relation. Uncertainty in asset returns may also contribute to a positive or negative relationship between risk aversion and savings, depending on the IES and lifecycle income profile. The overall impact of risk aversion is therefore theoretically ambiguous, with its sign depending on the magnitude of the different risks at play. Figure 1 shows, however, that RS agents save less than the "uncalibrated-additive" agents. Moreover, RS agents are less likely to purchase annuities: approximately 5% of them do so, compared to over 75% of the "uncalibrated-additive" agents. This is in line with the predictions of Proposition 1.

From a quantitative standpoint, this means that the effects of income and

[30] See Bommier and LeGrand (2019), who show, using risk-sensitive preferences in an infinite horizon setting, that there is a positive relationship between risk aversion and precautionary savings.

financial risks that we added in this quantitative investigation turn out to be too small to offset the effect of mortality risk highlighted in our theory section. The effect of the mortality risk tends therefore to dominate those of other risks. We can interpret this as an indication that mortality risks loom larger for individuals in their decision making than do the other risks they face.[31]

Figure 1 shows also that RS agents have a higher VSL at all ages and are less likely to invest in stocks than the "uncalibrated-additive" agents. This simply reflects that risk aversion increases the willingness to reduce exposure to mortality and financial risks.

Overall, our results regarding the impact of risk aversion highlight that the more risk averse are individuals, the more they dislike taking risks of any kind, whether they are related to mortality, income, or financial matters. It is worth noting, however, that our findings strongly contrast with those of well-known studies in the HF literature, such as Gomes and Michaelides (2005, 2008), who find a positive relationship between risk aversion and stock market participation, and Inkmann et al. (2011), who find that risk aversion increases the demand for annuities. Explanations for these differences are provided in Section 5.

Comparison of the calibrated models. Let us now compare the predictions of Baseline RS and Baseline Additive specifications. By construction, both specifications closely match accumulated wealth, stock market participation, and annuity market participation at age 65, and the same value of life at age 45. The lifecycle profiles for consumption, wealth, and stock market participation are therefore similar (as are portfolio shares held in each asset), though with a divergence in the VSL over the second half of the lifecycle.[32]

A fundamental difference between the specifications is in how they confront the 'annuity puzzle', that is, how they rationalize realistically low annuity demand. Low annuity demand is rationalized for additive agents with a counterfactually high administrative load (23% compared to 10% for the RS agents) and a relatively low time preference parameter (0.953 compared to 0.975 in the RS case). RS agents, on the other hand, are concerned that purchasing annuities may lead to a loss in case of an early death. Although they value the benefits of holding annuities to insure against the consumption needs in the case of a long life, they also want to retain significant investments in bonds or in stocks so that the early death adverse

[31] This fact is also reflected in the very high willingness to pay for mortality risk reduction revealed by empirical studies, which we used to calibrate our model.

[32] Increasing risk aversion leads to a greater willingness to pay to avoid dramatic outcomes, such as death at young age, as compared to adverse but less dramatic outcomes, such as death at old age. Increasing risk aversion therefore tends to amplify the relationship between age and VSL.

event is mitigated by the transmission of a bequest to her heirs. This allows a low level of annuity demand to be rationalized, even if annuities are priced at close to actuarially fair levels.

Sensitivity analysis. In Appendix C, we provide a sensitivity analysis where we investigate how our model matches some untargeted wealth moments and the sensitivity to the value of the VSL that we target. Here, we provide a brief discussion of the second of these – the impact of the VSL estimate. From our theoretical section, we have learned two things. First, the choice of a plausible (and hence positive) value of the VSL is central to the relationship between risk aversion and household choices. Second, when the VSL is positive and large enough, the risk-sensitive model is similar to an additive model with age-dependent time preference parameters. The precise value of the VSL therefore does not matter much, as long as it is large. A natural question, given that the empirical literature on VSL has not reached a consensus estimate of its value, is how much our results would be impacted by choosing a VSL target in the upper or lower range of empirical estimates; or, to phrase it differently, whether we are in the zone where the VSL is large enough for its exact value not to matter much.

The value that we choose, \$12.9m, in 2022 prices, corresponds, as we discussed above, to the 'central estimate' proposed by US Department of Health and Human Services (2016). To assess whether this particular choice is instrumental in the results, we estimate the parameters using the same calibration strategy and the same calibration targets as in Section 4.4, except that we consider two alternative targets for the VSL, reported in US Department of Health and Human Services (2016) as low and high estimates. These are, respectively \$6.1m and \$19.7m.

The calibrated parameters for the two VSL targets, as well as the plots of the related lifecycle profiles for consumption, wealth, VSL, and stock market participation can be found in Appendix C.1. These graphs show clearly that changing the VSL target has very little impact on individual decisions (consumption, wealth, asset market participation, and annuity purchase). The only significant difference between calibrations is on the VSL lifecycle profiles. The take-away is that, while choosing a much lower VSL target would noticeably impact our quantitative findings, the VSL target is sufficiently large for its precise value not to matter. We also deduce that our calibrated model is in fact relatively close to the limit model with infinite value of life that we presented in Section 3.2.

Additional quantitative experiments. In the Online Appendix, we provide additional quantitative experiments to illustrate how the various sources of risk end up affecting, quantitatively speaking, the impact of risk aversion. The aim of this exercise is to complement our theoretical model in Section 3, where mortality is the only risk in play. More precisely, we start with a simple model (labeled SM) with no income risk and no stock market participation and consider three values for the risk aversion parameter: $k = 0$ (additive model), baseline value of k ($k = 1.02$, as in our benchmark calibration), and a high value of k ($k = 1.25$, illustrating an increase in risk aversion). We subsequently add income risk (which leads to the simple model with income uncertainty labeled SMIU), and finally stock market participation (which yields the baseline model labeled BS). This allows us to investigate how these other sources of risk end up affecting the impact of risk aversion on decision-making. Detailed results about consumption profiles, wealth accumulation and annuity holdings can be found in the Online Appendix. Figure 2 below gathers those regarding annuity holdings. The results, including those reported in the Online Appendix, can be summarized as follows.

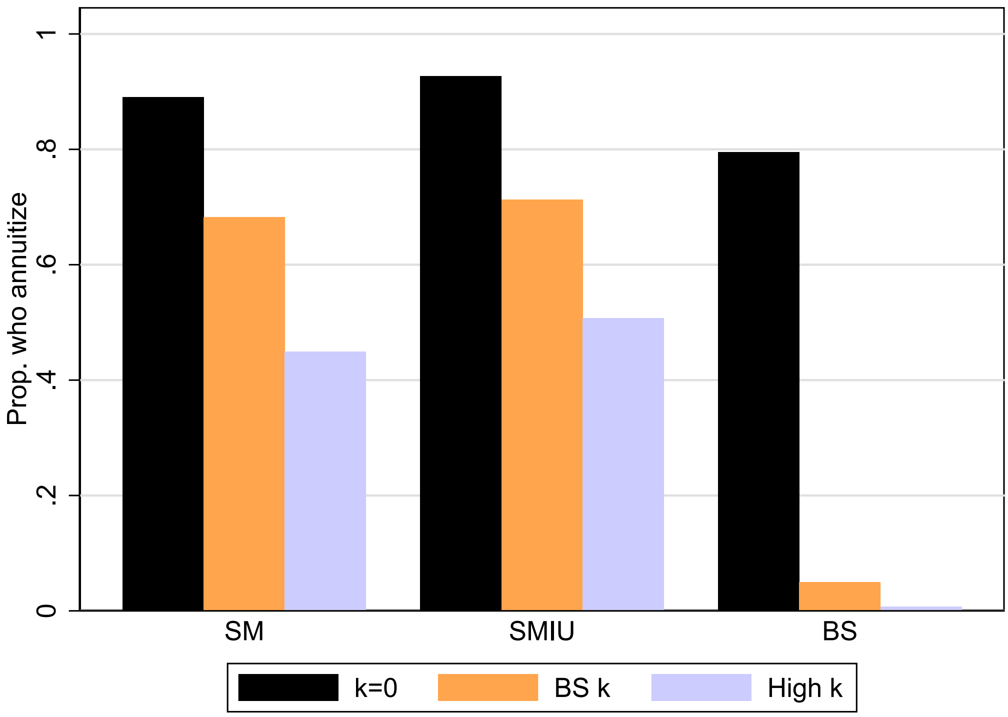
First, the findings for the model with mortality risk only are in line with those of the theory of Section 3. What was formally shown to hold for an infinite value of mortality risk reduction in Proposition 2 also tends to hold when assuming a finite value of mortality risk reduction: more risk averse agents tend to consume earlier in life and hence to save less. Regarding annuities, the share of agents holding annuities decreases with risk aversion (see Figure 2, below).

Second, the introduction of income risk confirms that the well-known precautionary motive is at play in our model. All agents tend to save more than in the absence of income risks, reflecting a standard prudence effect, and a higher proportion of agents hold annuities (see the SMIU case in Figure 2). The precautionary motive is stronger for more risk averse agents. However, the overall quantitative effect remains rather modest and when income and mortality risks are combined, more risk averse agents still end up saving less. As in the simple model with no income risk, risk aversion reduces the demand for annuities.

Finally, introducing stocks leads all agents to have a higher mean wealth, because of the higher returns offered by stocks. The benefit of risky assets is higher when agents participate more often in stock markets and hold a higher share of their wealth in stocks, which is the case for less risk averse agents. Thus, stock participation magnifies the wealth heterogeneity that would be implied by risk aversion heterogeneity.

The introduction of stocks also reduces the demand for annuities. The effect is stronger for more risk averse agents, as can be seen in Figure 2. This reflects the fact that risk-averse agents are reluctant to invest in annuities because of the risk of leaving a small bequest in the event of an early death (making the worst-case

Figure 2: Share of households holding annuities. SM refers to the model with mortality risk only; SMIU to the model with mortality and income risk; BS to the baseline model with the mortality, income and asset return risks. The three risk aversion calibrations are: $k = 0$ (additive model); baseline value of k ($k = 1.02$); and the high value of k ($k = 1.25$).



scenario even worse). They are then eager to find good substitutes for annuities (such as stocks and bonds) to finance consumption in old age. The availability of risky assets, which on average offer a higher return, reinforces this substitution effect.

Overall, two main lessons can be drawn from this exercise. First, the (negative) impact of risk aversion on savings is mainly driven by the presence of mortality risk and only marginally attenuated by the precautionary savings motive due to income risk. Second, the effect of risk aversion on annuity holdings is amplified when stock market participation is available.

5 Relation to previous studies using EZW preferences

Comparison with the HF literature. Most papers in the HF literature rely on the additive specification, which lacks the flexibility to fully study the role of risk aversion in decision-making over the lifecycle. In such papers, the expression "risk aversion" is most often used to refer to a parameter (σ in our paper) that governs both intertemporal substitutability and risk preferences. It is, however, well understood that models that assume different values for the IES are not comparable in terms of risk aversion (see Kihlstrom and Mirman, 1974, for instance). The findings of those papers, therefore, cannot be compared to ours.

The relevant comparison is with the subset of papers, such as Gomes and Michaelides (2005, 2008), Inkmann et al. (2011), and many others, which, like us, use recursive preferences to study the role of risk aversion in isolation. A key difference is that our model was designed to fit empirical estimates of the VSL, without imposing preference homotheticity, while these papers typically use EZW preferences to obtain homothetic (and tractable) specifications without considering the implications for the VSL. Formally, for EZW preferences, in the presence of bequests the recursion defining the utility conditional on being alive becomes:

$$V_t = \left((1-\beta) c_t^{1-\sigma} + \beta \left(E_t \left[\pi_t V_{t+1}^{1-\gamma} + (1-\pi_t)\theta x_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\sigma}{1-\gamma}} \right)^{\frac{1}{1-\sigma}}, \tag{26}$$

where, as in (24), θ determines the intensity of the bequest motive.[33] Most papers focus on the case where both σ and γ are greater than one, so that $\frac{1-\sigma}{1-\gamma} > 0$, which avoids the implausible time discounting patterns discussed in Section 2.2 and illustrated in the Online Appendix. There is typically no discussion of the value of mortality risk reduction in HF papers, as they assume that mortality is exogenous. Equation (26) nevertheless implicitly assumes a specific sign for the value of mortality risk reduction. In particular:

$$\frac{\partial V_t}{\partial \pi_t} = \beta \frac{E_t[V_{t+1}^{1-\gamma}] - \theta E_t[x_{t+1}^{1-\gamma}]}{1-\gamma} \left(E_t \left[\pi_t V_{t+1}^{1-\gamma} + (1-\pi_t)\theta x_{t+1}^{1-\gamma} \right] \right)^{\frac{\gamma-\sigma}{1-\gamma}} V_t^{\sigma},$$

which can be positive or negative. With $\gamma > 1$, a positive value of mortality risk reduction is obtained only if $\theta > \frac{E_t[V_{t+1}^{1-\gamma}]}{E_t[x_{t+1}^{1-\gamma}]}$. The results of Gomes and Michaelides (2005, 2008) and those of Inkmann et al. (2011) indicate that this condition does not hold (at least not always) in their simulations.[34] In particular, a negative value of mortality risk reduction is systematically obtained when there is no bequest

[33]A formal derivation of equation (26) can be found in the appendix of Gomes et al. (2009).

[34]One should notice, moreover, that if specification (26) were to be used with $\gamma > 1$ and a parameter θ large enough to generate positive values of mortality risk reduction, we would obtain a framework that is still non-monotone and where the intensity of the bequest motive would increase the willingness to pay for mortality risk reduction: $\frac{\partial^2 V_t}{\partial \pi_t \partial \theta} > 0$. However, this would go against intuition, since deriving utility from bequest reduces the welfare gap between life and death. In models such as the risk-sensitive or additive models, which this paper argues are better suited to studying decisions in the face of mortality risk, altruism has a negative impact on the value of mortality risk reduction (see the expressions of VSL in the Online Appendix for instance) – as one would expect.

motive ($\theta = 0$), a case considered in several instances in these papers. With a negative value of mortality risk reduction, risk aversion is found to amplify savings, in line with the theoretical predictions we derive in Section 3.1 for risk-sensitive preferences. This increase in saving behavior, in turn, generates differences in the propensity to pay the stock market participation cost. This explains why Gomes and Michaelides (2005, 2008) find that more risk averse agents tend to participate more frequently in the stock market. Moreover, with a negative value of mortality risk reduction, the risk of losing annuitized wealth in case of an early death is not seen as a major concern, as short lives are seen as good outcomes. This impacts the willingness to purchase annuities, which is found to increase with risk aversion (Inkmann et al., 2011), again in line with the theoretical results developed in Section 3 for risk-sensitive preferences.[35] Overall, in EZW models with a negative VSL, risk aversion is found to increase savings, stock market participation and annuity purchases (see also the Online Appendix for an illustration). These conclusions are opposite to those obtained in the quantitative exercise of Section 4, where we impose a positive VSL. We therefore see that considering a model featuring monotonicity and a positive VSL – as we do in the current contribution – sheds new light on the literature. If monotonicity is key to the ability to derive formal results as those of Section 3, Pashchenko and Porapakkarm (2022), who study the annuity demand with *non-homothetic* EZW preferences tend to indicate that adjusting EZW preferences to get a positive value of life would already lead to qualitative results that are in line with ours. However, the use of non-homothetic EZW specifications is questionable as they are no more tractable than risk-sensitive preferences and retain the non-monotonicity of the general class of EZW preferences.

Comparison with the VoL literature. In the VoL literature, the recursive models of Hugonnier et al. (2013) and Córdoba and Ripoll (2017) do not account for bequest motives and thus do not investigate the trade-offs between annuity and bond purchases. In order to obtain a positive VSL those papers focus on the case where $\gamma < 1$. When $\sigma < 1$ (i.e., when the IES is above one), the exponent $\frac{1-\sigma}{1-\gamma}$ is positive and savings are found to decrease with risk aversion, in line with our findings. When $\sigma > 1$, that is when the IES is below 1, Hugonnier et al. (2013) and Córdoba and Ripoll (2017) find that risk aversion increases savings. But as we explained in Section 2.2, this is because such models assume a discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ that decreases with the survival probability π_t (see the Online Appendix for a numerical illustration). Such a discount factor typically becomes much larger

[35] Note however, that formally speaking, the results obtained for risk-sensitive preferences – and the underlying intuitions – do not directly extend to EZW preferences, as the latter are not monotone with respect to first-order stochastic dominance.

than one at old ages, yielding counterfactual predictions.

Moreover, assuming a discount factor that is (systematically) larger than β, which is the case when $\frac{1-\sigma}{1-\gamma} < 0$, means that the agent will choose dominated strategies for her savings. Indeed, consider an agent who in period t does not know whether she will die at the end of period t or not. This agent foresees two possibilities: either she dies or she survives. If she were certain to die at the end of period t, she would consume all her wealth and save nothing because of the absence of bequest motives. If she were certain to survive, she would save as if $\pi_t = 1$, which implies a discount factor equal to β. By assuming a discount factor that is greater than β in the presence of mortality risk, the model predicts that the agent would save more than would be optimal in any of the two possibilities (survival or death). This saving choice is a dominated strategy, as saving less (e.g., saving the same amount as if she survives for sure) would make the agent better off whether she survives or not. The monotonicity breakdown becomes more extreme as π_t gets smaller. If $\pi_t \to 0$, meaning that period t is almost certainly the last period of life, then $\pi_t^{\frac{1-\sigma}{1-\gamma}}$ tends to ∞. In this case, the propensity to consume in period t tends to zero and the agent saves everything to consume in the next period, even though she will almost certainly not live to see it. Moreover, this tendency to oversave (relative to non-dominated strategies) is magnified when the risk aversion parameter γ increases toward one (while keeping $\sigma > 1$ and π_t fixed). Increasing risk aversion increases the extent to which dominated strategies are chosen. This drives the positive relationship between savings and risk aversion found in these papers when $\sigma > 1$.

6 Conclusion

Inspired by Samuelson (1937), economic contributions on intertemporal choice have most often relied on models that assume time-additive preferences. While time-additive preferences offered an elegant framework to formalize insightful theories, such as Modigliani's lifecycle hypothesis, they have some limitations. One of the caveats associated with the time-additive model is its lack of flexibility, in particular, the fact that its use means that risk aversion and intertemporal substitutability cannot be disentangled. This was underlined both by theoreticians (Epstein and Zin, 1989) and experimentalists (Andersen et al., 2008). Theoretical contributions, such as those of Epstein and Zin (1989) and Weil (1990), have addressed this limitation by introducing a recursive framework that is both tractable and flexible. These EZW preferences have met with a remarkable success. While EZW preferences were initially developed to deal with infinitely-lived agents, their adaptation to finite and random horizon settings leads to the serious difficulties that we discussed in Section 2.

In the current paper, we propose a framework that can model lifecycle behaviors with recursive preferences that are well-defined, flexible, monotone, and can simultaneously match realistic (positive) values for mortality risk reduction and plausible lifecycle profiles for consumption and wealth. We outline how using such a specification facilitates new and intuitive insights on the role of risk aversion, affording a potential explanation for several documented – though imperfectly understood – household behaviors, such as the low demand for annuities.

In an era of ongoing demographic changes, the economics of aging will remain a key research area, with new forms of risks becoming increasing sources of concern (e.g., the increased likelihood of expensive long-term care at the oldest ages). While time-additive preferences have facilitated valuable insights, we argue that the field would benefit from a framework that allows decisions in the face of survival risk to be studied in a setting where the role of risk aversion can be separated from that of intertemporal substitutability. The approach put forward in this paper could serve as a foundation of such a framework for both the HF and the VoL literatures.

Appendix

A Proof of Proposition 1

Note that at the optimum, we must have $b_k > 0$ and $w_0 - b_k - a_k > 0$. We focus on the case where $a_k > 0$. The first-order conditions of the consumption problem (11) yields after some arrangement:

$$b_k + \frac{a_k}{\pi_0} = b_k e^{-\frac{k(1-\beta)}{\sigma}(u(R^f b_k + R^f \frac{a_k}{\pi_0}) - v(R^f b_k))}, \tag{27}$$

$$\frac{w_0 - b_k - a_k}{b_k + \frac{a_k}{\pi_0}} = (\beta (R^f)^{1-\sigma})^{-\frac{1}{\sigma}} \Big(\pi_0 + (1-\pi_0) e^{k(1-\beta)(u(R^f b_k + R^f \frac{a_k}{\pi_0}) - v(R^f b_k))}\Big)^{\frac{1}{\sigma}}. \tag{28}$$

We define $a'_k = \frac{\partial a_k}{\partial k}$, $b'_k = \frac{\partial b_k}{\partial k}$, $\kappa = \frac{k(1-\beta)}{\sigma}(R^f)^{1-\sigma} > 0$, and $\Delta = k(1-\beta)(u(R^f b_k + R^f \frac{a_k}{\pi_0}) - v(R^f b_k))$ and obtain from (27) and (28):

$$\frac{b'_k + \frac{a'_k}{\pi_0}}{b_k + \frac{a_k}{\pi_0}} = \frac{b'_k}{b_k} - \kappa((b_k + \frac{a_k}{\pi_0})^{-\sigma}(b'_k + \frac{a'_k}{\pi_0}) - b_k^{-\sigma} b'_k) - \frac{\kappa}{k}\Delta, \tag{29}$$

$$\frac{b'_k + a'_k}{w_0 - b_k - a_k} + \frac{b'_k + \frac{a'_k}{\pi_0}}{b_k + \frac{a_k}{\pi_0}} = \frac{\Big(\kappa(b_k^{-\sigma} b'_k - (b_k + \frac{a_k}{\pi_0})^{-\sigma}(b'_k + \frac{a'_k}{\pi_0})) + \frac{\kappa}{k}\Delta\Big)(1-\pi_0)e^{\Delta}}{\pi_0 + (1-\pi_0)e^{\Delta}}. \tag{30}$$

Equations (29) and (30) yield:

$$b'_k = -\lambda_{b'_k} a'_k, \tag{31}$$

$$\text{with: } \lambda_{b'_k} = \frac{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{1}{\pi_0 + (1-\pi_0)e^{\Delta}}}{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{\pi_0}{\pi_0 + (1-\pi_0)e^{\Delta}} + \frac{1}{b_k} \frac{(1-\pi_0)e^{\Delta}}{\pi_0 + (1-\pi_0)e^{\Delta}}}, \tag{32}$$

which is positive for all values of Δ. We obtain from (31):

$$a'_k + b'_k = \frac{\frac{1}{b_k(b_k + \frac{a_k}{\pi_0})}\Big(\frac{a_k}{\pi_0} e^{\Delta} + b_k(e^{\Delta} - 1)\Big)\frac{(1-\pi_0)}{\pi_0 + (1-\pi_0)e^{\Delta}}}{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{\pi_0}{\pi_0 + (1-\pi_0)e^{\Delta}} + \frac{1}{b_k} \frac{(1-\pi_0)e^{\Delta}}{\pi_0 + (1-\pi_0)e^{\Delta}}} a'_k. \tag{33}$$

We also obtain from (31):

$$\frac{a'_k}{\pi_0} + b'_k = \lambda_{\frac{a'_k}{\pi_0} + b'_k} a'_k, \tag{34}$$

$$\lambda_{\frac{a'_k}{\pi_0} + b'_k} = \frac{\frac{1}{w_0 - b_k - a_k} \frac{1-\pi_0}{\pi_0} + \frac{1}{b_k} \frac{(1-\pi_0)e^{\Delta}}{\pi_0 + (1-\pi_0)e^{\Delta}} \frac{1}{\pi_0}}{\frac{1}{w_0 - b_k - a_k} + \frac{1}{b_k + \frac{a_k}{\pi_0}} \frac{\pi_0}{\pi_0 + (1-\pi_0)e^{\Delta}} + \frac{1}{b_k} \frac{(1-\pi_0)e^{\Delta}}{\pi_0 + (1-\pi_0)e^{\Delta}}} > 0.$$

Denoting by $U' = \frac{\partial}{\partial k}(\frac{c_{0,k}^{1-\sigma}}{1-\sigma} + \beta\frac{c_{1,k}^{1-\sigma}}{1-\sigma})$, we obtain using (28), (33), and (34):

$$\frac{U'}{c_0^{-\sigma}} = \frac{\frac{1}{w_0-b_k-a_k}\frac{1-\pi_0}{\pi_0} + \frac{1}{b_k}\frac{(1-\pi_0)^2e^{2\Delta}}{\pi_0(\pi_0+(1-\pi_0)e^{\Delta})} + \frac{1}{b_k+\frac{a_k}{\pi_0}}\frac{(1-\pi_0)e^{\Delta}}{\pi_0+(1-\pi_0)e^{\Delta}}}{\frac{1}{w_0-b_k-a_k} + \frac{1}{b_k+\frac{a_k}{\pi_0}}\frac{\pi_0}{\pi_0+(1-\pi_0)e^{\Delta}} + \frac{1}{b_k}\frac{(1-\pi_0)e^{\Delta}}{\pi_0+(1-\pi_0)e^{\Delta}}}a_k'. \tag{35}$$

We therefore deduce from (31), (33), (34), and (35), that $-\frac{\partial b_k}{\partial k}$, $-\frac{\partial c_{0,k}}{\partial k}$, $\frac{\partial c_{1,k}}{\partial k}$, and $\frac{\partial}{\partial k}(u(c_{0,k}) + \beta u(c_{1,k}))$ have the same sign as $\frac{\partial a_k}{\partial k}$. Using (31) and (34) with (29), we obtain after substitution:

$$\left(\frac{\lambda_{b_k'+\frac{a_k'}{\pi_0}}}{b_k+\frac{a_k}{\pi_0}} + \kappa\frac{\lambda_{b_k'+\frac{a_k'}{\pi_0}}}{(b_k+\frac{a_k}{\pi_0})^{\sigma}} + \frac{\lambda_{b_k'}}{b_k} + \kappa\frac{\lambda_{b_k'}}{b_k^{\sigma}}\right)a_k' = -\frac{\kappa}{k}\Delta, \tag{36}$$

which implies that $\frac{\partial a_k}{\partial k}$ has the same sign as $-\Delta$ (when $k > 0$). Since the sign of Δ is the same as the sign of $\frac{\partial V_0}{\partial \pi_0}$, this concludes the proof.

B Calibration details

B.1 Estimation of Earnings Process

To estimate the earnings process we use data from the Panel Study of Income Dynamics (following Moffitt and Zhang, 2018). The population that we set out to study is men born in the 1940, a cohort for which data over much of their lifecycle is available. Our focus on men is due to the fact that, for those born in the 1940s, labor market trajectories for women were more frequently interrupted than in the present day, and we do not wish these (often planned) transitions in wages to appear as due to the realization of risk. We make a number of other restrictions following the approach in Moffitt and Zhang (2018).[36] We use data from 1969 to 2015 on those aged between 18 and 65 and those who reported working in the previous year and reported positive wage earnings.

To estimate the deterministic component of wages we regress the log of real earnings ($\ln y$) on a cubic in age (t), dummies for decade of birth (D_j), and the annual unemployment rate (where U_t denotes the unemployment rate in the year an individual was aged t). The earnings equation is:

$$\ln y_{i,t} = \delta_0 + \delta_1 t + \delta_2 t^2 + \sum_{j=0}^{9}\delta_{1900+10j}D_{1900+10j} + \delta_3 U_t + \epsilon_{it} \tag{37}$$

where ϵ_{it} contains a persistent stochastic component of earnings and an *iid* mea-

[36]In particular we keep only sample members who report that they are household heads, drop full time students, and those who are not part of specific PSID sub-samples (Latino and SEO).

surement error and is discussed below. We estimate the parameters of this earnings equation using OLS with the PSID sample weights. We then use the estimated age polynomial, the estimated cohort dummy for those born in the 1940s, and the mean unemployment rate over our period to estimate a deterministic wage profile $(\mu_t)_t$, given below, and reported in Figure 3:

$$\ln \mu_t = \hat{\delta}_0 + \hat{\delta}_1 t + \hat{\delta}_2 t^2 + \hat{\delta}_{1940} D_{1940} + \hat{\delta}_3 \bar{U} \tag{38}$$

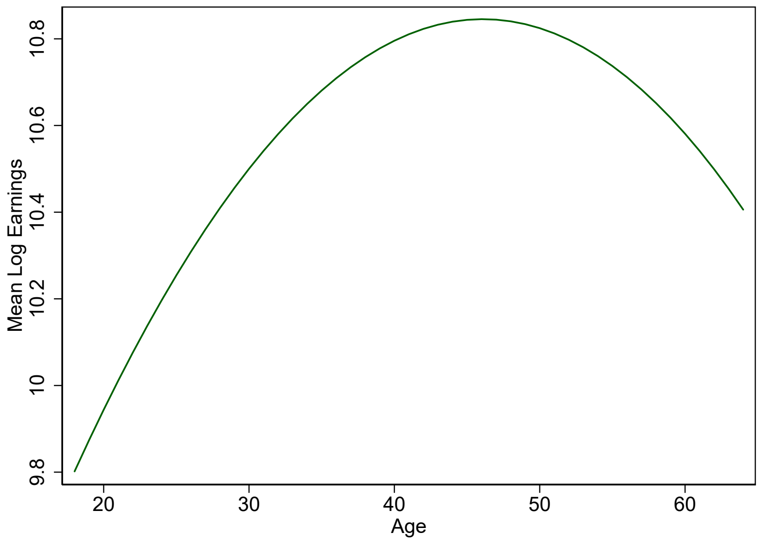


Figure 3: Earnings Process

The stochastic component of wages, represented by ϵ_{it} in equation (37), contains an AR(1) term, ζ_t, and an *iid* component, assumed to be measurement error:

$$\epsilon_{it} = \zeta_{i,t} + \nu_{i,t},$$
$$\zeta_{i,t} = \rho\zeta_{i,t-1} + \upsilon_{i,t},$$

where $\upsilon_t \sim \mathcal{N}(0, \sigma_\upsilon^2)$ and $\eta_{it} \sim \mathcal{N}(\mu, \sigma_\eta^2)$. To estimate the parameters of this system, we first obtain estimated residuals from the estimated version of (37). We then obtain the empirical auto-covariance matrix of these residuals and estimate the ρ, σ_υ^2, and σ_υ^2 using standard minimum distance method (see for example, Guvenen, 2009 or Low et al., 2010). We use data only up to 1996, after this year the PSID changed to being a biennial survey, complicating its use in the estimation of annual earnings processes. We exclude from estimation any variances or auto-covariances calculated with fewer than 10 observations. For the weighting matrix in the implementation of the minimum distance estimation, we use the inverse of the diagonal of the empirical auto-covariance matrix. This weighting matrix avoids the small-sample bias associated with using efficient weighting (see

Altonji and Segal, 1996). This yields estimates of ρ, σ_v^2, and σ_υ^2 of 0.977, 0.010, and 0.131 respectively.

B.2 Wealth and Stock Market Participation

To estimate the wealth target, we use the Health and Retirement Study. In keeping with our aim to study the behavior of one cohort we take those born in the 1940s. Our model is a model of single individuals – so we restrict ourselves to individuals whom we observe as not married from the fourth wave of the Health and Retirement Study (i.e., they are either never married or, if divorced or widowed, some time has elapsed since their separation or bereavement). There is a trade-off between the homogeneity of the population we study and sample size: even when we pool observations across waves, we have 430 observations on individuals between the ages of 60 and 69. We calculate median wealth in the years when individuals are between the ages of 60 and 69. Median wealth (using the HRS sampling weights) is approximately \$230,000. We use this as a calibration target and match it to the simulated median wealth at age 65.

To estimate the proportion of the sample that participates in equity markets, we take the same sample we used to calculate median wealth. We compute the proportion of the cohort who reported holding stocks in wave 10 of the data in 2010 or earlier. This is 65%.

B.3 Bequest parameters

To calibrate bequest parameters, we compute the marginal propensity to consume (MPC) and the maximum wealth at which no bequest will be left for an agent in the last period before certain death, when she can only invest in a riskless bond. Up to a factor $(1-\beta)^{-1}$, her program is: $\max_{b\in\mathbb{R}}\ u(W-b)+\beta v(R^f b)$. There are two cases. (i) No bequest is left iff $u'(W)\geq \beta R^f v'(0)$. (ii) Otherwise, there is a positive bequest, determined by: $u'(W-b)=\beta R^f v'(R^f b)$, which implies that the maximal no-bequest wealth, denoted by W_0, solves $u'(W_0)=\beta R^f v'(0)$, or using the expressions (23) and (24) for instantaneous utility functions:

$$W_0 = \kappa\overline{x},\ \text{with}\ \kappa = \left(\beta R^f \theta\right)^{-\frac{1}{\sigma}}. \tag{39}$$

If $W \geq W_0$, the optimal bequest b and MPC are given by:

$$b = \frac{W-\kappa\overline{x}}{1+\kappa R^f} \quad \text{and} \quad \frac{\partial c}{\partial W} = \frac{R^f\kappa}{1+R^f\kappa}. \tag{40}$$

C Sensitivity analysis

C.1 High and low targets for the VSL

We calibrate the model using the same targets as in the baseline calibration, except that we consider in turn, a high VSL target of 19.7 million USD and a low VSL target of 6.1 million. We report in Table 4 the value of parameters whose

Table 4: Parameter calibration with low and high VSL targets

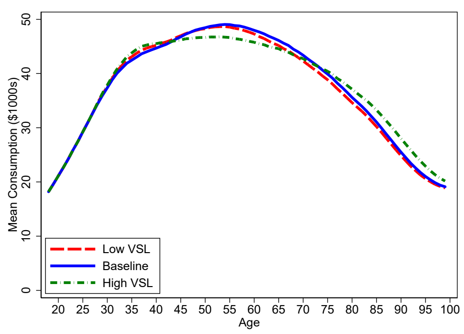
Parameter	Low VSL		High VSL	
	RS	Additive	RS	Additive
Risk aversion parameter, k	0.94	$0.00^{\dagger}$	1.01	$0.00^{\dagger}$
Life-death utility gap, u_l	2.64	7.46	5.67	24.08
Discount factor, β	0.963	0.953	0.981	0.953
Annuity admin. load, δ	$10\%^{\dagger}$	23%	$10\%^{\dagger}$	23%
Participation cost, F	106% of $\bar{y}$	123% of $\bar{y}$	117% of $\bar{y}$	123% of $\bar{y}$

Notes: One unit of consumption is equal to $\bar{y}$. Quantities indicated by a † are imposed rather than estimated.

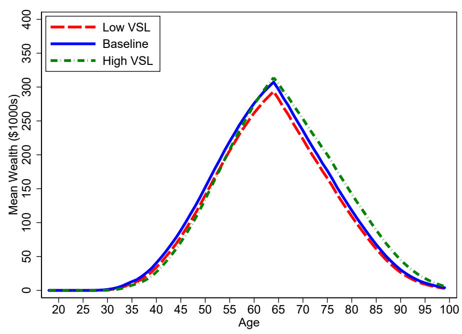
calibration is impacted compared to the baseline. Individual lifetime profiles are almost unchanged, as can be seen in Figure 4.

Figure 4: Lifecycle profiles for different VSL targets

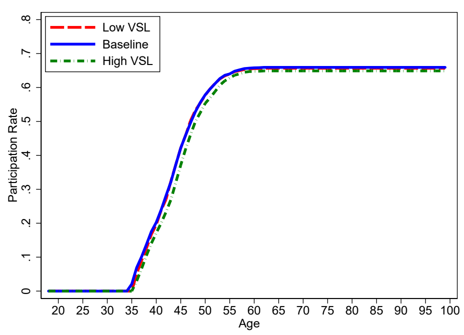
(a) Consumption



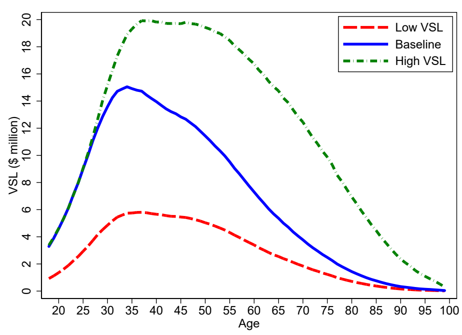
(b) Median wealth



(c) Stock market participation



(d) VSL



C.2 Comparison to untargetted wealth moment

Figure 5 takes Figure 1b and adds two additional (untargeted) wealth moments: median wealth for the cohort we study pooling across ages 51 to 55 (such that the mean age is 53.5) and across 56 and 59 (such that the mean age is 57.5). These are, respectively, approximately $113,000 and $187,000. The match between our simulated profiles and the untargetted moments is close.

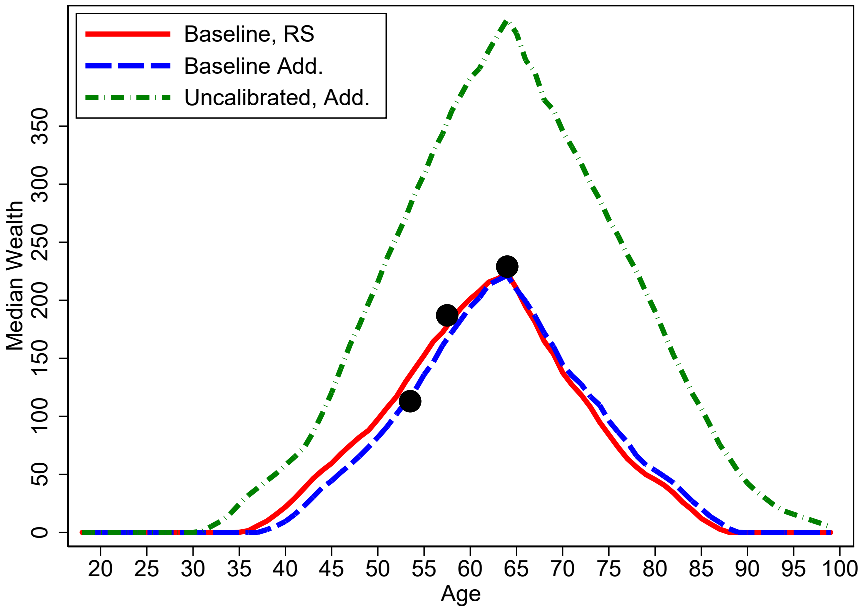


Figure 5: Showing other wealth targets

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