# Online Appendix to "Recursive Preferences, the Value of Life, and Household Finance"

June 10, 2026

This online appendix is organized as follows:

1. Section S.1 complements Section 2.2 by illustrating, first, the difficulties that may arise when using EZW preferences in the context of uncertain lifetimes and, second, the advantages of using risk-sensitive (RS) preferences. We focus on a simple setting in which there is no bequest motive and no annuities. We find that, unless the IES is greater than 1 and the risk-aversion parameter is less than 1, EZW preferences generate either implausible consumption paths or implausible values for mortality risk reduction. In contrast, RS preferences are found to provide reasonable consumption and values of mortality risk reduction in all cases we consider. Moreover we obtain responses to changes in risk aversion that reproduce the qualitative mortality-risk mechanism of Section 3: when the value of mortality-risk reduction is positive, higher risk aversion leads to front-loaded consumption and lower savings.

2. Section S.2 considers a series of computational experiments to better understand how the different risks of the model (mortality, income and asset return) interact with each other and contribute to our explanation of the relationships between risk aversion on the one hand and savings and annuity holdings on the other.

3. Section S.3 contains two proofs: (i) the proof of Proposition 2, which is close to the proof of Proposition 1 provided in the main text, and (ii) the proof of translation invariance of RS preferences. It also contains two other technical developments: (iii) the derivation of the VSL expressions in the RS and additive setups, and (iv) the computation of the limits of the EZW model when IES or risk aversion parameter converges to 1.

4. Section S.4 presents the computational method used to solve the quantitative model.

5. Section S.5 describes the model extension to health risk, including its calibration and outcomes.

To avoid confusion in the numbering of equations and sections between the main text and this supplemental online appendix, all numbers in this online appendix are prefixed by “S”. Conversely, numbers without a prefix refer to an equation or a section in the main text.

# S.1 Comparing consumption and VSL profiles with EZW and RS preferences

To complement the discussion in Sections 2.2 and 2.3 of the limitations of EZW preferences and the case for RS preferences, we examine the predictions each framework delivers in a simple setting with no bequest motive and no annuities. For each preference type we document how the predictions change as we vary the inverse-IES parameter $\sigma$ and the risk-aversion parameter ($\gamma$ in EZW and $k$ in RS). We focus on the two dimensions where the choice of recursion is especially consequential: the consumption profile and the value of life. The aim is to assess whether each recursion can deliver plausible consumption and VSL profiles under empirically relevant variation in the IES and risk aversion, and to compare the two frameworks on as equal a footing as possible.

Section S.1.1 shows that homothetic EZW preferences yield plausible life-cycle behavior only in the corner of the parameter space where both $\sigma < 1$ and $\gamma < 1$, and that this corner requires an IES above one that the empirical literature does not support. By contrast, Section S.1.2 shows that RS preferences deliver plausible consumption and a positive, well-shaped VSL across both IES values we consider, including the $\sigma = 2$ region that the IES evidence favors and that EZW cannot accommodate.

## S.1.1 Using EZW preferences

In the absence of bequest motives, the standard homothetic EZW specification corresponds to the following recursion:

$$V_t = \left((1-\beta)c_t^{1-\sigma} + \beta\pi_t^{\frac{1-\sigma}{1-\gamma}} \left(E[V_{t+1}^{1-\gamma}]\right)^{\frac{1-\sigma}{1-\gamma}}\right)^{\frac{1}{1-\sigma}}, \tag{S.1}$$

where $0 < \sigma \neq 1$ is the inverse of the IES and $0 < \gamma \neq 1$ is the risk aversion parameter (see Gomes and Michaelides, 2005 or Córdoba and Ripoll, 2017 among others).[1]

[1] We ignore the cases with $\sigma = 1$, corresponding to additive or RS preferences that are already discussed in the main text. As explained in Section S.3.4 below, the cases with $\gamma = 1$ but $\sigma \neq 1$

As discussed in the main text, when $\frac{1-\sigma}{1-\gamma} < 0$, the recursion (S.1) may be ill-defined and admits $V_t = 0$ (independently of consumption choices) as the unique solution. A further discussion of the theoretical aspects can be found in Bommier et al. (2021).

Table S.1: EZW calibration.

| Parameter | Value | Source |
|---|---|---|
| *Demographics* | | |
| Retirement date, $T_R$ | 47 (= 65 − 18) | SSA Historical Normal Retirement Age in US |
| Maximal life duration, $T_M$ | 82 (= 100 − 18) | |
| Cond. survival rates, $\{\pi_t\}$ | | US Life Tables for a male cohort born in 1940 (Bell and Miller, 2005) |
| *Endowments* | | |
| Average wage, $\bar{y}$ | US$ 43,104 | Life-cycle average for 1940s men (own estimates) |
| Age productivity, $\{\mu_t\}$ | | Life-cycle average for 1940s men (own estimates) |
| Public pension, $y^R$ | 40% $\times \bar{y}$ | Average SS replacement rate (Biggs and Springstead, 2008) |
| Labor income autocorr., $\rho$ | 0.977 | Own estimates |
| Var. of persistent shocks, $\sigma_\upsilon^2$ | 0.010 | Own estimates |
| *Asset Markets* | | |
| Gross risk-free return, $R^f$ | 1.02 | Campbell and Viceira (2002) |
| Equity premium, $\omega$ | 4% | Campbell and Viceira (2002) |
| Stock volatility, $\sigma_\nu$ | 15.7% | Campbell and Viceira (2002) |
| Participation cost, $F$ | 123% of $\bar{y}$ | Own calibration (additive model) |
| *Preferences* | | |
| Inverse of IES, $\sigma$ | 2 | Own calibration (additive model) |
| Discount factor, $\beta$ | 0.953 | Own calibration (additive model) |
| Life-death utility gap, $u_l$ | 0 | Homothetic preferences |
| Bequest motive strength, $\theta$ | 0 | No bequest motive |
| Bequest luxury good, $\overline{x}$ | 0 | No bequest motive |

*Notes:* One unit of consumption is equal to $\bar{y}$.

Here, we ignore these convergence issues and focus on the model implications for

are not well defined.

consumption and savings paths, as well as for the value of mortality risk reduction.[2] In this exercise, we make the following assumptions:

- the demographic aspects (mortality risk, retirement age, maximum age) are the same as in the main text;
- the labor income is risky and follows the same calibration as in the main text;
- agents can invest in riskless bonds and risky stocks, whose returns are the same as in the main text; there is no annuity;
- participation in the stock market is subject to a once-in-a-life cost, which is the same as in the main text;
- the time preference parameter is set to 0.953, as in the calibration for the additive model in our main analysis;
- there is no bequest motive: $\theta = \bar{x} = 0$.

We summarize the calibration in Table S.1. As in the main text, the reported values assume that 1 unit of consumption corresponds to the average income $\bar{y}$.

EZW specifications represented as in (S.1) can be grouped into four ($= 2 \times 2$) main categories depending on whether $\sigma$ is less than or greater than 1 and whether $\gamma$ is less than or greater than 1. Whether $\sigma \lesseqgtr 1$ and $\gamma \lesseqgtr 1$ matters for two reasons. First, it determines the sign of the exponent $\frac{1-\sigma}{1-\gamma}$, and thus how the discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ behaves at old ages when $\pi_t$ becomes small. Second, it also determines the sign of $\frac{\partial V_t}{\partial \pi_t}$, and hence the sign of the value of mortality risk reduction. This can be summarized in Table S.2.

Table S.2: Discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ and marginal utility of survival probability $\frac{\partial V_t}{\partial \pi_t}$ in the EZW model as a function of parameter values.

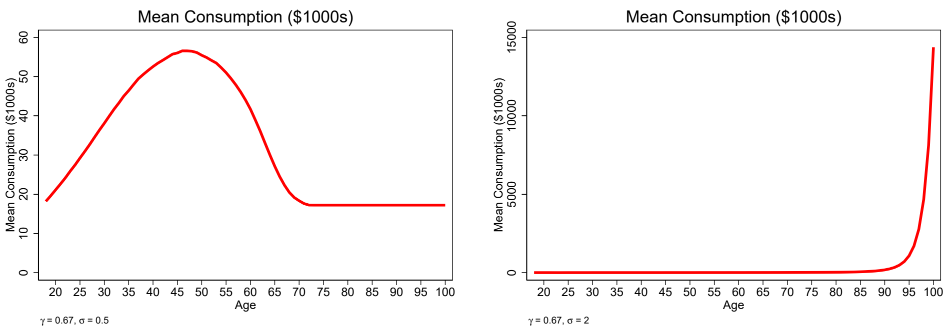
| | $\sigma < 1$ | $\sigma > 1$ |
|---|---|---|
| $\gamma < 1$ | $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} < 1$<br>$\frac{\partial V_t}{\partial \pi_t} > 0$ | $\boldsymbol{\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} \gg 1}$ **for small** $\boldsymbol{\pi_t}$<br>$\frac{\partial V_t}{\partial \pi_t} > 0$ |
| $\gamma > 1$ | $\boldsymbol{\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} \gg 1}$ **for small** $\boldsymbol{\pi_t}$<br>$\boldsymbol{\frac{\partial V_t}{\partial \pi_t} < 0}$ | $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}} < 1$<br>$\boldsymbol{\frac{\partial V_t}{\partial \pi_t} < 0}$ |

[2]Formally, if $\frac{1-\sigma}{1-\gamma} < 0$, we will assume that at the final date $T_{\max}$ (such that $\pi_{T_{\max}} = 0$) – which corresponds to 100 years in our data – we have $V_{T_{\max}} = (1-\beta)^{\frac{1}{1-\sigma}} c_{T_{\max}}$, while formally accounting for convergence issues would lead to $V_{T_{\max}} = 0$ (and thus $V_t = 0$ for all $t \leq T_{\max}$). This assumption is equivalent to assuming that the discount rate is set to zero in the final period.

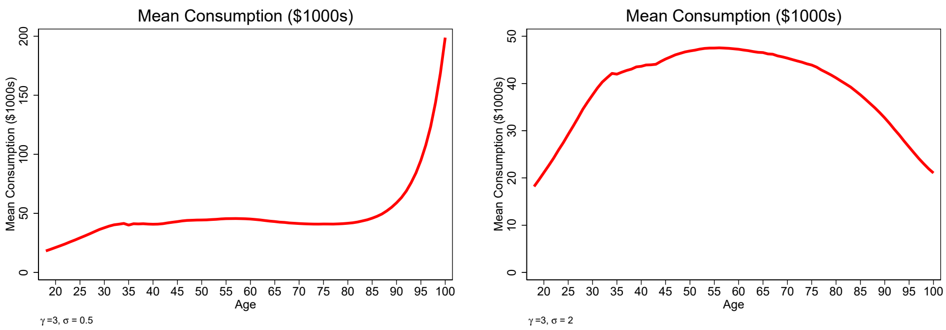
In Table S.2, we have bolded and shaded the properties that we consider to be major issues. The shaded entries correspond either to an effective discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ that becomes very large when $\pi_t$ is small (i.e., at old ages), or to the sign of the derivative $\frac{\partial V_t}{\partial \pi_t}$ that implies a negative value of mortality risk reduction. Table S.2 shows that every case except the one where both $\sigma$ and $\gamma$ are smaller than 1 has at least one of these problems: either an implausible patience pattern at old ages, a negative value of mortality risk reduction, or both. To illustrate these issues, we plot the consumption and VSL profiles for the four combinations of $\sigma \lesseqgtr 1$ and $\gamma \lesseqgtr 1$. For the inverse of IES, we use $\sigma = \frac{1}{2}$ and $\sigma = 2$, while for the risk aversion parameter, we use $\gamma = \frac{2}{3}$ and $\gamma = 3$.[3]

Figure S.1: Life-cycle consumption paths for the four parametrizations of the EZW model.

Case (a): $\sigma = \frac{1}{2}$ and $\gamma = \frac{2}{3}$ — Case (b): $\sigma = 2$ and $\gamma = \frac{2}{3}$




Case (c): $\sigma = \frac{1}{2}$ and $\gamma = 3$ — Case (d): $\sigma = 2$ and $\gamma = 3$



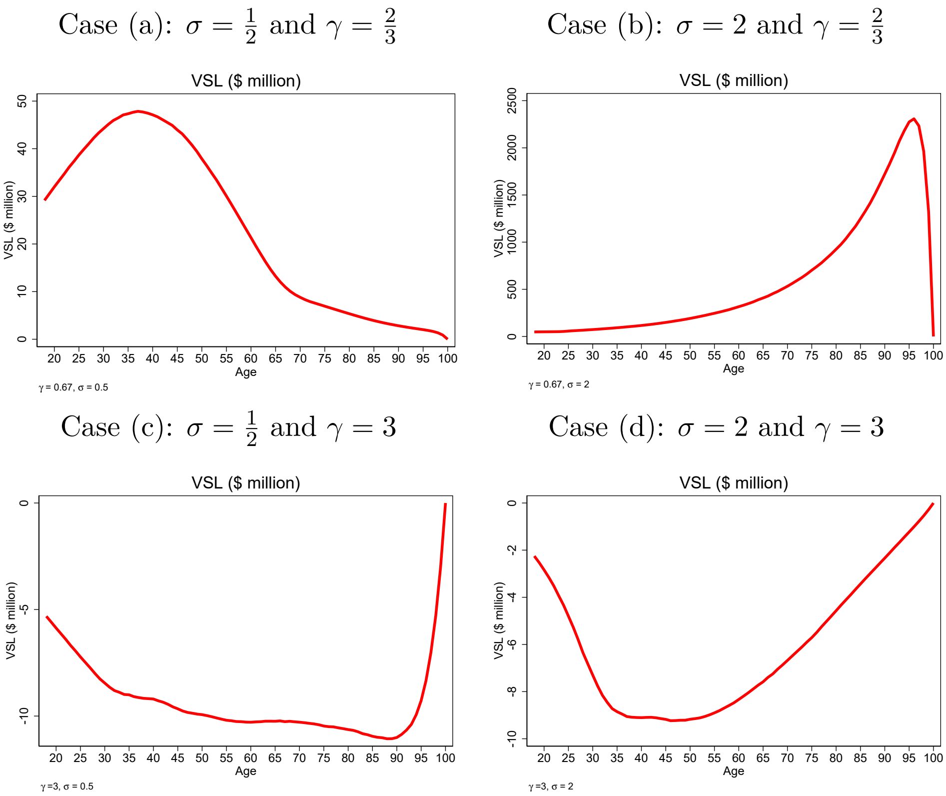

The life-cycle consumption paths are plotted in Figure S.1. Panels (b) and (c) show counterfactual consumption profiles, in which consumption remains low for most of the life-cycle, before rising sharply in old age. This is due to the fact that the parameters in panels (b) and (c) imply $\frac{1-\sigma}{1-\gamma} < 0$ and thus a discount factor $\beta\pi_t^{\frac{1-\sigma}{1-\gamma}}$ that becomes large (and much larger than one) at old ages, when $\pi_t$ is small. In these two specifications, mortality increases patience, which makes them

[3]We choose these values so that $\frac{1-\sigma}{1-\gamma}$ is never equal to 1, in which case the EZW model reduces to the additive model.

unsuitable for modeling consumption and saving behavior. Cases (a) and (d) do not suffer from these consumption-profile drawbacks, because the effective discount factor becomes small at old ages.

However, case (d), where $\gamma$ and $\sigma$ are both greater than one, predicts counterfactual negative values of mortality risk reduction. This can be seen in Figure S.2, where we plot the life-cycle VSL path for all four cases. In case (d), VSL is negative at all ages, meaning that people would be willing to pay to shorten their lives. This VSL is negative whenever $\gamma > 1$, and so the same is true of case (c). Cases (b) and (c) display, in addition, implausible VSL paths, which mirror the counterfactual consumption paths already discussed. Only case (a), where both $\gamma$ and $\sigma$ are below one, delivers a positive, plausibly shaped VSL. Note that, even in the context of exogenous mortality, assuming a negative value of life has problematic consequences. Specifically, as discussed in Section 3 of the main paper, the sign of the value of life determines the impact of risk aversion.

Figure S.2: Life-cycle VSL paths for four parametrizations of the EZW model.




Case (a): $\sigma = \frac{1}{2}$ and $\gamma = \frac{2}{3}$

Case (b): $\sigma = 2$ and $\gamma = \frac{2}{3}$

Case (c): $\sigma = \frac{1}{2}$ and $\gamma = 3$

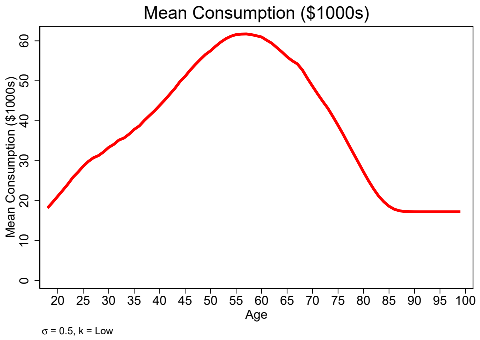
Case (d): $\sigma = 2$ and $\gamma = 3$

### S.1.2 Using RS preferences

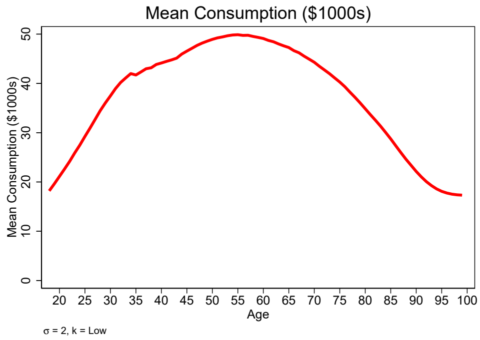
The EZW exercise leaves only the corner with $\sigma < 1$ and $\gamma < 1$ as a candidate region (case (a)). A natural question is whether RS preferences, subjected to an

Figure S.3: Life-cycle consumption paths for the four parametrizations of the RS model.

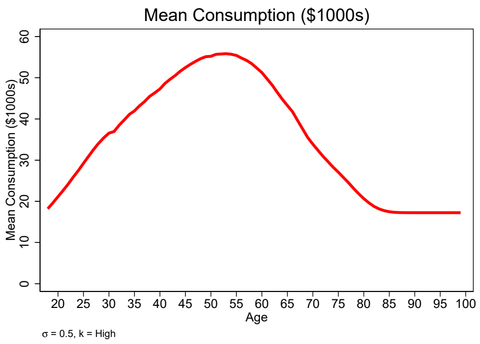
Case (a): $\sigma = \frac{1}{2}$ and $k = k^{\text{Base}}_{\sigma=0.5}$



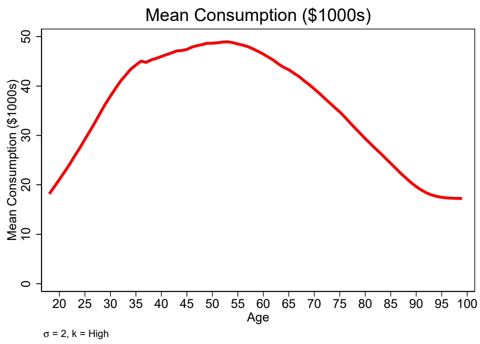

Case (b): $\sigma = 2$ and $k = k^{\text{Base}}_{\sigma=2}$




Case (c): $\sigma = \frac{1}{2}$ and $k = k^{\text{Perturbed}}_{\sigma=0.5}$




Case (d): $\sigma = 2$ and $k = k^{\text{Perturbed}}_{\sigma=2}$




analogous exercise, fare any better. We show that they do across both of the IES values we consider, including the $\sigma = 2$ (IES $= 0.5$) region where EZW can yield counterfactual predictions. RS preferences deliver plausible consumption profiles, a positive value of life, and comparative statics that align with the theory of Section 3. To make the comparison as close to like-for-like as possible, as for the EZW exercise above, we turn off annuities and bequests, and retain the same demographics, labor income process, asset returns, and stock market participation structure. The only substantive difference is the preference recursion: RS preferences in place of EZW.

The two exercises do, however, differ in how the parameters are chosen. The four EZW cases were generated by varying the two preference parameters $\sigma$ and $\gamma$ while borrowing all remaining parameters from the additive calibration, since it is not possible to target all our empirical objects with EZW preferences. With RS preferences, it is possible to do so: for each value of $\sigma \in \{0.5, 2\}$ we choose the four parameters $k, \beta, F$, and $u_l$ to have a close match with VSL at age 45, median wealth at age 65, and the stock market participation rate at age 65. We denote as $k^{\text{Base}}_{\sigma=0.5}$ and $k^{\text{Base}}_{\sigma=2}$ the values of $k$ in each case.[4] Having calibrated the model for each

[4] $k_{\sigma=2}$ differs from the baseline value of $k$ as in this model there are no bequests or annuities.

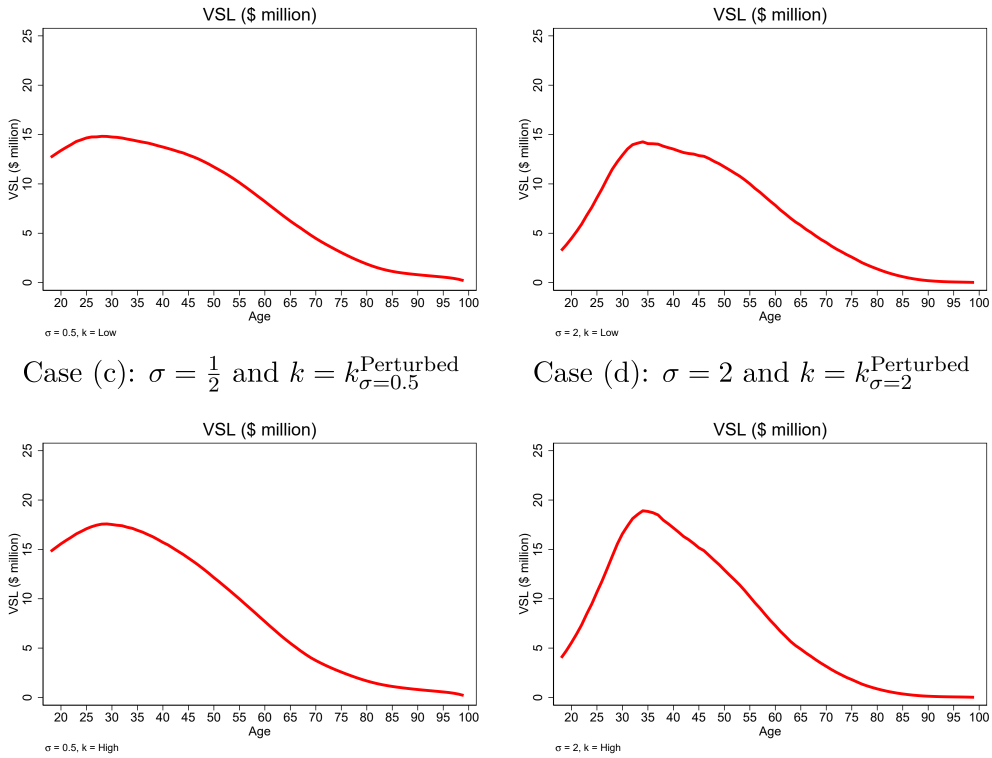
value of $\sigma$, we then trace out the role of risk aversion by increasing $k$ while holding the calibration otherwise fixed.[5] We denote these as $k_{\sigma=0.5}^{\text{Perturbed}}$ and $k_{\sigma=2}^{\text{Perturbed}}$. We consider consumption and the VSL in turn, the two dimensions on which the EZW specifications ran into difficulty.

**Consumption.** Figure S.3 plots mean consumption for both $\sigma = 0.5$ and $\sigma = 2$, at the baseline and perturbed values of $k$. In all four panels consumption follows a plausible hump-shaped profile, rising through working life and declining in retirement. The contrast with EZW is direct. At $\sigma = 2$, EZW generated consumption that either stayed low for most of life before exploding at old age (case (b)) or, while plausible in shape, was accompanied by a negative value of life (case (d)). RS preferences deliver a sensible consumption profile at $\sigma = 2$ throughout. At $\sigma = 0.5$, RS behaves much as the well-behaved EZW corner does (case (a)). The gain from RS is thus concentrated at $\sigma = 2$, precisely where EZW fails.

Figure S.4: Life-cycle VSL paths for the four parametrizations of the RS model.

Case (a): $\sigma = \frac{1}{2}$ and $k = k_{\sigma=0.5}^{\text{Base}}$ Case (b): $\sigma = 2$ and $k = k_{\sigma=2}^{\text{Base}}$

Case (c): $\sigma = \frac{1}{2}$ and $k = k_{\sigma=0.5}^{\text{Perturbed}}$ Case (d): $\sigma = 2$ and $k = k_{\sigma=2}^{\text{Perturbed}}$




**Value of life.** Figure S.4 reports the corresponding VSL profiles. In every panel the VSL is positive at all ages and hump-shaped, peaking in mid-life and

[5] We increase $k$ by the same factor (22%) that we use later in this Appendix.

declining into old age, which is the empirically sensible pattern. In EZW a positive VSL requires $\gamma < 1$ and $\sigma < 1$ to avoid the old-age consumption explosion. RS preferences deliver positive, well-behaved VSL profiles at both IES values, including the $\sigma = 2$ case that the empirical IES literature favors and that EZW cannot accommodate.

**Summary.** In summary, the numerical results above show that homothetic EZW preferences avoid the two main pathologies documented in this appendix, exploding old-age consumption and a negative value of life, only in the region where $\sigma < 1$ and $\gamma < 1$. This is a restrictive corner of the parameter space, as it requires an IES above one, whereas much of the empirical evidence points to an IES below one. RS preferences do not impose the same constraint: risk aversion and the IES can be set freely, guided by the evidence rather than by the need to avoid counterfactual predictions.

## S.2 Understanding the interplay between the different risks

We conduct a series of computational experiments to better understand how the risks in the model shape the role of risk aversion in savings and annuity holdings under RS preferences.

We consider three different risk environments. In the first one, agents face solely a mortality risk. There is no income risk or stock market participation. This simple model is a direct extension of our theoretical framework of Section 3 to the case of multiple periods and a finite value of mortality risk reduction. The second environment adds income risk to the previous setup. The last environment further allows stock market participation and therefore corresponds to our baseline model.

We investigate the role of risk aversion both in the small (variations between two positive values of $k$) and in the large (between $k = 0$ and $k > 0$). We therefore consider three values for the risk aversion parameter: $k = 0$ (additive model), baseline value of $k$ ($k = 1.02$, as in our benchmark calibration), and a high value of $k$ ($k = 1.25$, illustrating an increase in risk aversion).

The calibration is the same as in the baseline calibration, except that we shut off one or both of two sources of risk: income risk and/or stock holdings. The value of the higher $k$ is chosen such that the consumption profiles of the three models intersect (i.e., such that there is one age for which the consumption is the same in all models). The results can be summarized as follows.

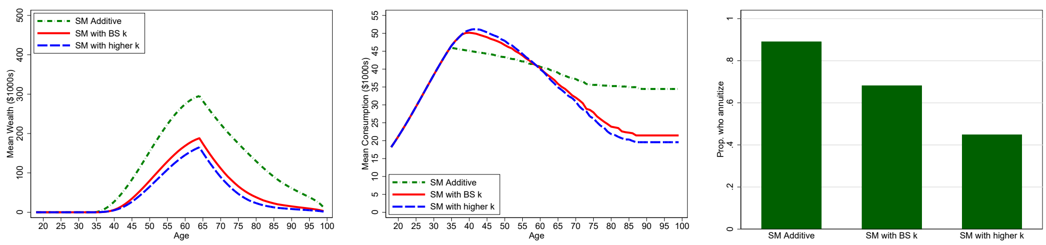
**Case 1: There is no income uncertainty and no risky asset.** We shut off the income risk and participation in the stock market. In the calibration, this implies setting income risk variances to zero and the stock market participation cost to infinity. Agents are heterogeneous in terms of income paths, just like when we assume income uncertainty, but they know ex-ante their (deterministic) income trajectories. They can only save in riskless bonds and annuities. The results are gathered in Figure S.5, where we report the life-cycle profiles related to savings and consumption (Figures S.5a and S.5b), as well as the proportion of agents holding annuities (S.5c). The acronym SM in graph labels stands for "simple model".

Risk aversion, whether varied locally or from the additive benchmark, has an unambiguous effect on savings and consumption. More risk averse agents tend to consume earlier in life and hence to save less. Conversely, they tend to consume less in late life. At early ages, consumption and savings are the same for the three models because young agents are credit-constrained and simply live hand-to-mouth. Because income is the same in all three models, the consumption profiles when credit-constrained are the same in the three models. Regarding annuities, the share of agents holding annuities decreases with risk aversion.

These results are in line with the theory of Section 3. The result formally shown in Proposition 2 for an infinite value of mortality risk reduction extends qualitatively to a finite value of mortality risk reduction.

Figure S.5: Simple Model.

(a) Total Wealth. (b) Consumption. (c) Annuity.



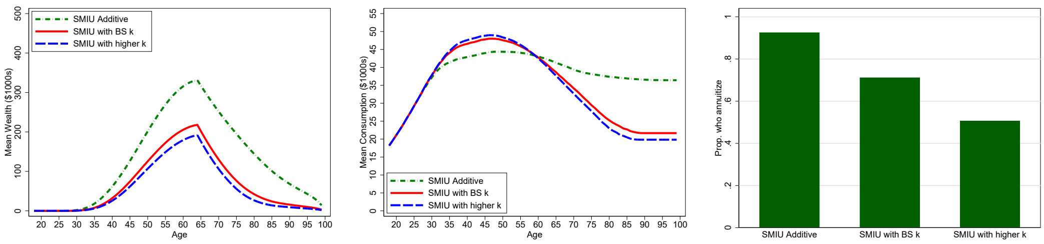

**Case 2: There is no risky asset but there is realistically calibrated income risk.** We add income risk to the previous model. Stock market participation cost is still infinite, but income risk variances are set to their baseline values. The results are gathered in Figure S.6, where we report the life-cycle profiles related to savings and consumption (Figures S.6a and S.6b), as well as the proportion of agents holding annuities (S.6c).

The overall picture remains similar to the previous case. However, because of income uncertainty, agents in the three cases tend to save more than in the absence of income risk, reflecting a standard prudence effect. Hence, agents stop being constrained at slightly younger ages. The precautionary motive also tends to increase the proportion of agents holding annuities, and the effect is stronger for more risk averse agents. In other words, the cross-effect of income risk and risk aversion on savings is positive.[6]

This case confirms that the well-known precautionary motive is at play in our model. However, its quantitative effect is rather modest and it does not offset the effect of mortality risk on savings. When income and mortality risks are combined, more risk averse agents still end up saving less. As in Case 1, annuity purchases fall with risk aversion.

Figure S.6: Simple Model with Income Uncertainty.

(a) Total Wealth. (b) Consumption. (c) Annuity.




**Case 3: There is a risky asset and income risk.** This case corresponds to the baseline calibration (hence, the label BS). Results are plotted in Figure S.7. The introduction of risky assets has an unambiguous effect on savings: all agents tend to have a higher mean wealth. Since risky assets offer on average a much higher return than riskless assets, participation in stock markets allows agents, through compounding effects, to accumulate higher savings. The benefit of risky assets is higher when agents participate more often in stock markets and hold a higher share of their wealth in stocks. Both effects are more pronounced for less risk averse agents. This explains why additive agents see the largest change in wealth with the introduction of risky assets, while agents with higher $k$ see the smallest change in wealth from it. The differences between life-cycle savings profiles in Figure S.7a are thus much more pronounced than in Figure S.6a in the absence of risky assets.

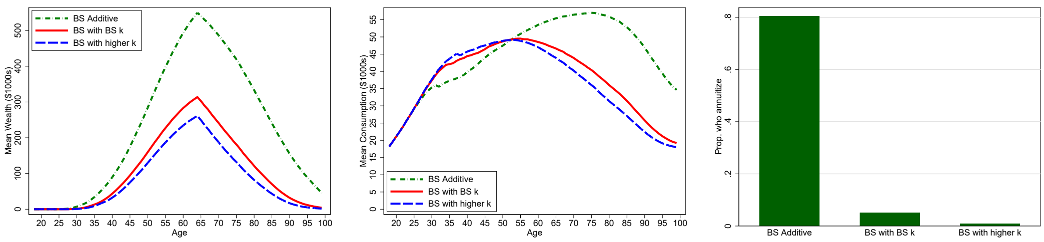
In addition to magnifying wealth heterogeneity generated by risk aversion heterogeneity, the introduction of risky assets also strongly impacts the demand for annuities. For all agents, annuities are crowded out by stocks. The introduction of

[6]This cross-effect is not clear cut on total savings of Figure S.5a because of the compounded wealth effects.

stocks reduces the demand for annuities. As can be seen in Figure S.7c, the effect is stronger for more risk averse agents. The reduction in the proportion of agents annuitizing is modest for additive agents, as the share decreases by approximately 10 points compared to the SMIU. The reduction is much larger for risk averse agents (with baseline $k$) for whom the share of annuity holdings drops from around 70% to 5%. The crowding out effect of stocks on annuities is thus very sizable for risk averse agents. Finally, the most risk averse agents hold very few annuities.

Figure S.7: Baseline Model.

(a) Total Wealth. (b) Consumption. (c) Annuity.




# S.3 Additional proofs

This section contains two proofs and two technical developments. First, in Section S.3.1, we prove the result of Proposition 2. Second, in Section S.3.2, we prove the translation invariance property of RS preferences. Third, in Section S.3.3, we provide the detailed derivation of the VSL expressions in the RS and additive setups. Finally, in Section S.3.4, we discuss the limit cases where the IES or the risk aversion tends to 1 in EZW specifications.

## S.3.1 Proof of Proposition 2

The proposition extends the two-period comparative statics to the finite-horizon limit in which the value of mortality-risk reduction is infinite. The objective is to show that an increase in $\kappa$ lowers annuity holdings, raises bond holdings, and lowers total saving at each date: $a'_{\kappa,t} < 0$, $b'_{\kappa,t} > 0$, and $a'_{\kappa,t} + b'_{\kappa,t} < 0$. The proof proceeds by backward induction.

We consider the program of an agent choosing riskless savings and annuities to maximize her utility function given by the limit of the RS utility function for $u_l \to \infty$, while holding the quantity $\kappa = ku_l$ constant. The program can be written as follows:

$$V_t^\infty(a_{t-1}, b_{t-1}) = \max_{a_t, b_t} (1-\beta)u(y_t + R^f(b_{t-1} + \frac{a_{t-1}}{\pi_{t-1}}) - a_t - b_t) \tag{S.2}$$
$$+ \frac{\beta}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} \Big(\pi_t V_{t+1}^\infty(a_t, b_t) + (1-\pi_t)z_{\kappa,t+1}E_t[(1-\beta)v(R^f b_t)]\Big),$$

where $(y_t)_t$ is the deterministic income path and we highlight the dependence of $z_{\kappa,t}$ in $\kappa$:

$$\log(z_{\kappa,t}) = (1-\beta)\kappa - \beta \log\left(\pi_t/z_{\kappa,t+1} + 1 - \pi_t\right). \tag{S.3}$$

We assume that the program admits an interior solution at all dates, denoted by $(a_{\kappa,t}, b_{\kappa,t})_t$. These solutions are given by the following first-order conditions:

$$\begin{aligned}
&u'(y_t + R^f(b_{\kappa,t-1} + \frac{a_{\kappa,t-1}}{\pi_{t-1}}) - a_{\kappa,t} - b_{\kappa,t}) \\
&= \frac{\beta R^f}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} \Bigg(\pi_t u'(y_{t+1} + R^f(b_{\kappa,t} + \frac{a_{\kappa,t}}{\pi_t}) - a_{\kappa,t+1} - b_{\kappa,t+1}) \\
&\qquad + (1-\pi_t)z_{\kappa,t+1}v'(R^f b_{\kappa,t})\Bigg), \\
&= \frac{\beta R^f}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} u'(y_{t+1} + R^f(b_{\kappa,t} + \frac{a_{\kappa,t}}{\pi_t}) - a_{\kappa,t+1} - b_{\kappa,t+1}),
\end{aligned}$$

or

$$\begin{aligned}
u'(y_t + R^f(b_{\kappa,t-1} + \frac{a_{\kappa,t-1}}{\pi_{t-1}}) - a_{\kappa,t} - b_{\kappa,t}) &= \frac{\beta R^f}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} \\
&\quad \times u'(y_{t+1} + R^f(b_{\kappa,t} + \frac{a_{\kappa,t}}{\pi_t}) - a_{\kappa,t+1} - b_{\kappa,t+1}), \\
u'(y_{t+1} + R^f(b_{\kappa,t} + \frac{a_{\kappa,t}}{\pi_t}) - a_{\kappa,t+1} - b_{\kappa,t+1}) &= z_{\kappa,t+1}v'(R^f b_{\kappa,t}), \\
u'(y_t + R^f(b_{\kappa,t-1} + \frac{a_{\kappa,t-1}}{\pi_{t-1}}) - a_{\kappa,t} - b_{\kappa,t}) &= \frac{\beta R^f z_{\kappa,t+1}}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} v'(R^f b_{\kappa,t}).
\end{aligned}$$

We compute the derivatives of the FOCs with respect to $\kappa$. We denote the partial derivatives by a prime: $a'_{\kappa,t} = \frac{\partial a_{\kappa,t}}{\partial \kappa}$, $b'_{\kappa,t} = \frac{\partial b_{\kappa,t}}{\partial \kappa}$, and $z'_{\kappa,t} = \frac{\partial z_{\kappa,t}}{\partial \kappa}$. We also denote by $c_{\kappa,t} = y_t + R^f(b_{\kappa,t-1} + \frac{a_{\kappa,t-1}}{\pi_{t-1}}) - a_{\kappa,t} - b_{\kappa,t}$ the date-$t$ consumption level. We obtain:

$$-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})}(b'_{\kappa,t} + a'_{\kappa,t}) = -\frac{(1-\pi_t)z'_{\kappa,t+1}}{\pi_t + (1-\pi_t)z_{\kappa,t+1}} \tag{S.4}$$
$$+ \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})}(R^f(b'_{\kappa,t} + \frac{a'_{\kappa,t}}{\pi_t}) - (b'_{\kappa,t+1} + a'_{\kappa,t+1})),$$

$$-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})}(b'_{\kappa,t} + a'_{\kappa,t}) = \frac{\pi_t z'_{\kappa,t+1}}{z_{\kappa,t+1}(\pi_t + (1-\pi_t)z_{\kappa,t+1})} + R^f \frac{v''(R^f b_{\kappa,t})}{v'(R^f b_{\kappa,t})} b'_{\kappa,t}. \tag{S.5}$$

We prove by backward induction that $a'_{\kappa,t} < 0$, $b'_{\kappa,t} > 0$, and $b'_{\kappa,t} + a'_{\kappa,t} < 0$.

At date $t = T_{\max} - 1$, we have $b'_{\kappa,t+1} = a'_{\kappa,t+1} = 0$ and the result follows from the two-period model. In particular, we have $b'_{\kappa,T_{\max}-1} + a'_{\kappa,T_{\max}-1} < 0$. We assume that the result holds at some date $t+1$, such that $b'_{\kappa,t+1} + a'_{\kappa,t+1} < 0$.

Combining (S.4) and (S.5) to remove $z'_{\kappa,t+1}$, we obtain:

$$b'_{\kappa,t} = -\lambda_{b'_{\kappa,t}} a'_{\kappa,t} + \mu_{b'_{\kappa,t}} (b'_{\kappa,t+1} + a'_{\kappa,t+1}), \tag{S.6}$$

$$\text{where: } \lambda_{b'_{\kappa,t}} = \frac{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})}}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t) z_{\kappa,t+1} R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}} > 0, \tag{S.7}$$

$$\mu_{b'_{\kappa,t}} = \frac{-\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} \frac{\pi_t}{\pi_t + (1-\pi_t) z_{\kappa,t+1}}}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t) z_{\kappa,t+1} R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}} > 0. \tag{S.8}$$

We then use (S.6) and (S.7) to obtain:

$$\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t} = \lambda_{\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t}} a'_{\kappa,t} + \mu_{b'_{\kappa,t}} (b'_{\kappa,t+1} + a'_{\kappa,t+1}), \tag{S.9}$$

where:

$$\lambda_{\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t}} = \frac{\frac{1-\pi_t}{\pi_t} \left( -\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{z_{\kappa,t+1} R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})} \right)}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t) z_{\kappa,t+1} R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}} > 0. \tag{S.10}$$

Combining (S.9) and (S.10) yields:

$$R^f (b'_{\kappa,t} + \frac{a'_{\kappa,t}}{\pi_t}) - (b'_{\kappa,t+1} + a'_{\kappa,t+1}) = R^f \lambda_{\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t}} a'_{\kappa,t} - \mu_{c'_{\kappa,t}} (b'_{\kappa,t+1} + a'_{\kappa,t+1}), \tag{S.11}$$

where:

$$\mu_{c'_{\kappa,t}} = \frac{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{(1-\pi_t) z_{\kappa,t+1} R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t) z_{\kappa,t+1} R^f}{\pi_t + (1-\pi_t) z_{\kappa,t+1}} \frac{v''(x_{t+1})}{v'(x_{t+1})}} > 0. \tag{S.12}$$

The difference of (S.4) and (S.5) implies:

$$0 = \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} (R^f (b'_{\kappa,t} + \frac{a'_{\kappa,t}}{\pi_t}) - (b'_{\kappa,t+1} + a'_{\kappa,t+1})) - \frac{z'_{\kappa,t+1}}{z_{\kappa,t+1}} - R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} b'_{\kappa,t},$$

which using (S.11) leads to:

$$\begin{aligned} a'_{\kappa,t} = & \frac{-\frac{z'_{\kappa,t+1}}{z_{\kappa,t+1}}}{-\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} R^f \lambda_{\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t}} - R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} \lambda_{b'_{\kappa,t}}} \\ & + \frac{-\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} \mu_{c'_{\kappa,t}} - R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} \mu_{b'_{\kappa,t}}}{-\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} R^f \lambda_{\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t}} - R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} \lambda_{b'_{\kappa,t}}} (b'_{\kappa,t+1} + a'_{\kappa,t+1}). \end{aligned} \tag{S.13}$$

Equation (S.3) implies that $\frac{z'_{\kappa,t+1}}{z_{\kappa,t+1}} > 0$. Thus, using the induction hypothesis, we have from (S.13) that $a'_{\kappa,t} < 0$ and from (S.11) that $\frac{a'_{\kappa,t}}{\pi_t} + b'_{\kappa,t} < 0$.

We also have from (S.6):

$$a'_{\kappa,t} + b'_{\kappa,t} = \frac{-\frac{(\pi_t-1)R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}\frac{v''(x_{t+1})}{v'(x_{t+1})}}{-\frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})} - \frac{\pi_t R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}\frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})} - \frac{(1-\pi_t)z_{\kappa,t+1}R^f}{\pi_t+(1-\pi_t)z_{\kappa,t+1}}\frac{v''(x_{t+1})}{v'(x_{t+1})}} a'_{\kappa,t}$$
$$- \frac{u''(c_{\kappa,t+1})}{u'(c_{\kappa,t+1})}\pi_t(b'_{\kappa,t+1} + a'_{\kappa,t+1}),$$

which with $a'_{\kappa,t} < 0$ implies $a'_{\kappa,t} + b'_{\kappa,t} < 0$.

We also have from (S.5) and (S.13):

$$-R^f \frac{v''(x_{t+1})}{v'(x_{t+1})} b'_{\kappa,t} = \frac{u''(c_{\kappa,t})}{u'(c_{\kappa,t})}(b'_{\kappa,t} + a'_{\kappa,t}) + \frac{\pi_t z'_{\kappa,t+1}}{z_{\kappa,t+1}(\pi_t + (1-\pi_t)z_{\kappa,t+1})} > 0,$$

implying $b'_{\kappa,t} > 0$.

### S.3.2 Proof of translation invariance in the RS model

We recall that the recursion characterizing the utility function representing RS preferences in the presence of bequest can be written as follows:

$$V_t = (1-\beta)u(c_t) - \frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-kV_{t+1}}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]\right), \quad \text{(S.14)}$$

where we use the same notation as in the main text.

We will prove the following lemma.

**Lemma 1** *Consider the specification (S.14) and assume that $v(0)$ is finite. Simultaneously changing the instantaneous utility function for consumption to $u(c) - (1-\beta)v(0)$ and that for bequest to $v(x) - v(0)$ produces a simple shift in utilities ($V_t$ is changed to $V_t - (1-\beta)v(0)$) that has no effect on preferences.*

In other words, Lemma 1 implies that if the utility of dying and leaving no bequest is finite, we can normalize $v$ by setting $v(0) = 0$ without loss of generality. Note that the results also hold when there is no bequest motive ($v(x)$ equals a finite constant $u_d$ independent from $x$).

**Proof.** Observe that we have:

$$\begin{aligned}
&\log\left(\pi_t E_t\left[e^{-kV_{t+1}}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]\right)\\
&= \log\left(e^{-k(1-\beta)v(0)}\left(\pi_t E_t\left[e^{-k(V_{t+1}-(1-\beta)v(0))}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right)\right),\\
&= -k(1-\beta)v(0) + \log\left(\pi_t E_t\left[e^{-k(V_{t+1}-(1-\beta)v(0))}\right] + (1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right).
\end{aligned}$$

Hence, we deduce:

$$
\begin{aligned}
V_t-(1-\beta)v(0) &= (1-\beta)u(c_t)+\beta(1-\beta)v(0)-(1-\beta)v(0)\\
&-\frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-k(V_{t+1}-(1-\beta)v(0))}\right]+(1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right),\\
&=(1-\beta)(u(c_t)-(1-\beta)v(0))\\
&-\frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-k(V_{t+1}-(1-\beta)v(0))}\right]+(1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right).
\end{aligned}
$$

Defining $\tilde{V}_t = V_t-(1-\beta)v(0)$ and $\tilde{u}(c)=u(c)-(1-\beta)v(0)$, we obtain that the preferences represented by $V_t$ may also be represented by $\tilde{V}_t$ defined by the following recursion:

$$
\tilde{V}_t=(1-\beta)\tilde{u}(c_t)-\frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-k\tilde{V}_{t+1}}\right]+(1-\pi_t)E_t\left[e^{-k(1-\beta)(v(x_{t+1})-v(0))}\right]\right),
$$

which completes the proof. ■

### S.3.3 Deriving VSL expressions

We denote by $w_t=A_{t-1}+R^f b_{t-1}+R^s_t s_{t-1}$ the beginning-of-period wealth by $\omega_t=q_t a_t+b_t+s_t$ the total saving choice, by $\alpha^b_t=\frac{b_t}{\omega_t}$ the share in bonds and by $\alpha^s_t=\frac{s_t}{\omega_t}$ the share in stocks. The program of the alive agent can be rewritten as:

$$
\begin{aligned}
V_t(w_t,A_{t-1},\eta_{t-1},\zeta_{t-1}) = &\max_{c_t\geq 0,\omega_t\geq 0,(\alpha^b_t,\alpha^s_t)\in[0,1]^2}(1-\beta)u(c_t)\\
&-\frac{\beta}{k}\log\left(\pi_t E_t\left[e^{-kV_{t+1}(w_{t+1},A_t,\eta_t,\zeta_t)}\right]+(1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]\right),
\end{aligned}
$$

subject to: $y_t+w_t=c_t+\omega_t+1_{\eta_t=1}1_{\eta_{t-1}=0}F$, $w_t=A_{t-1}+\omega_t(\frac{1}{q_t}+(R^f-\frac{1}{q_t})\alpha^b_t+(R^s_{t+1}-\frac{1}{q_t})\alpha^s_t)$, and $x_t=\omega_t(R^f\alpha^b_t+R^s_{t+1}\alpha^s_t)$.

The envelope theorem yields $\frac{\partial V_t}{\partial w_t}=\frac{\partial V_t}{\partial c_t}=(1-\beta)u'(c_t)$. Using $\frac{\partial(1/q_t)}{\partial \pi_t}=-\frac{1}{\pi_t q_t}$ and $\frac{\omega_t}{q_t}(1-\alpha^b_t-\alpha^s_t)=a_t$, after some manipulation, we get:

$$
\begin{aligned}
VSL_t=&\frac{\beta}{1-\beta}\frac{1}{c_t^{-\sigma}}\frac{\left(-\frac{E_t\left[e^{-kV_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}\right]-E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]}{k}\right)}{\pi_t E_t\left[e^{-kV_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}\right]+(1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]}\\
&-\beta a_t\frac{E_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}e^{-kV_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}\right]}{\pi_t E_t\left[e^{-kV_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}\right]+(1-\pi_t)E_t\left[e^{-k(1-\beta)v(x_{t+1})}\right]},
\end{aligned}
\tag{S.15}
$$

and in the additive case, by continuity for $k\to 0$:

$$
VSL^{add}_t=\frac{\beta}{c_t^{-\sigma}}\left(\frac{V^{add}_{t+1}(b_t,A_t,s_t,\eta_t,\zeta_t)}{1-\beta}-v(x_{t+1})\right)-\beta a_t E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\right].\tag{S.16}
$$

### S.3.4 The limit cases in the EZW model

We consider the no-bequest case. The recursion defining the utility $V_t$ representing EZW preferences is:

$$V_t = \left((1-\beta)c_t^{1-\sigma} + \beta\pi_t^{\frac{1-\sigma}{1-\gamma}}\left(E_t[V_{t+1}^{1-\gamma}]\right)^{\frac{1-\sigma}{1-\gamma}}\right)^{\frac{1}{1-\sigma}}. \tag{S.17}$$

We consider the limits of the utility function for $\sigma \to 1$ and $\gamma \to 1$.

#### S.3.4.1 The limit $\sigma \to 1$, while $\gamma \neq 1$.

We conduct a first-order development to compute the limit when $\sigma \to 1$. Taking the log of (S.17), we deduce that preferences can be represented by $\tilde{V}_t = \log V_t$ defined by the following recursion:

$$\tilde{V}_t = (1-\beta)\log(c_t) + \frac{\beta}{1-\gamma}\log(\pi_t E_t[e^{(1-\gamma)\tilde{V}_{t+1}}]), \tag{S.18}$$

which corresponds to the RS utility recursion with $k = \gamma - 1$ and $ku_d = \infty$. Note that recursion (S.18) requires $\pi_t > 0$ at all dates. Otherwise, if $\pi_{T_{\max}} = 0$ for some $T_{\max}$, then $(1-\gamma)\tilde{V}_t = -\infty$ for all $t \leq T_{\max}$.

#### S.3.4.2 The limit $\gamma \to 1$, while $\sigma \neq 1$.

This limit case is either undefined, degenerate or exhibits an implausible impatience pattern. To show that one may focus on the case where the only uncertainty is related to survival, so that the expectation sign $E_t$ can be removed in (S.17). The recursion (S.17) then becomes:

$$V_t = \left((1-\beta)c_t^{1-\sigma} + \beta\pi_t^{\frac{1-\sigma}{1-\gamma}} V_{t+1}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

To discuss the limit, one has to distinguish the cases where $\gamma$ tends to 1 from above or from below, and whether $\sigma$ is larger or smaller than 1.

In the case where $\gamma \to 1^-$ and $\sigma < 1$ then $\frac{1-\sigma}{1-\gamma} \to +\infty$ and $V_t \to (1-\beta)^{\frac{1}{1-\sigma}} c_t$. The model then exhibit infinite impatience, where the future does not matter. The case where $\gamma \to 1^+$ and $\sigma > 1$ is similar. When $\gamma \to 1^+$ and $\sigma < 1$ then $\frac{1-\sigma}{1-\gamma} \to -\infty$ and $V_t \to \infty$, the limit being then undefined. Last, when $\gamma \to 1^-$ and $\sigma > 1$ then $\frac{1-\sigma}{1-\gamma} \to -\infty$ and $V_t \to 0$, the limit model being a degenerate one.

#### S.3.4.3 The limit $\gamma \to 1$ and $\sigma \to 1$.

Due to the issues that appear when taking the limit $\gamma \to 1$ , the results is sensitive to the order in which the limits $\gamma \to 1$ and $\sigma \to 1$ are considered. However, the limit utility is always ill-defined, being equal to either 0 or infinity, independently of consumption choices.

# S.4 Details on the computational implementation

There is no analytical solution to the agents' problem outlined in Section 4 of the main paper. We solve the model and obtain decision rules numerically and then use those decision rules to simulate the agents' behavior. The next two subsections describe the solution of the model and the simulation of decision rules.

## S.4.1 Model solution

While alive, the agent maximizes her intertemporal utility by choosing a feasible allocation $(c_t, b_t, a_t, s_t, \eta_t)_{t\geq 0}$ in the set of feasible allocations, denoted $\mathcal{A}$. The utility $V_t$ of the alive agent at age $t$ depends on five state variables: the beginning-of-period holdings in bonds $b_{t-1}$, annuities $A_{t-1}$ and stocks $s_{t-1}$; the stochastic component of labor income, $\zeta_{t-1}$; and the stock market participation status, $\eta_{t-1} \in \{0, 1\}$. The last of these is discrete, while the first four are continuous. Given that annuity purchase may only occur in period $T_R - 1$, we have $A_t = 0$ for all $t < T_R$ and $A_t = a_{T_R-1}$ for $t \geq T_R$. Since there exists a maximal age for the agent, $T_M$, we solve the model by iterating on the value function, starting from the last period of life. Utility maximization involves solving:

$$V_{T_M}(b_{T_M-1}, A_{T_M-1}, s_{T_M-1}, \eta_{T_M-1}, \zeta_{T_M-1}) = \max_{(c_{T_M}, b_{T_M}, a_{T_M}, s_{T_M}, w_{T_M}, \eta_{T_M}) \in \mathcal{A}} (1-\beta)u(c_{T_M}) - \frac{\beta}{k} \log E_t \left[e^{-k(1-\beta)v(x_{T_M+1})}\right], \tag{S.19}$$

subject to the constraints, which we don't restate here, outlined in Section 4 of the main paper. Due to the presence of stocks in bequest, the continuation utility in case of death is uncertain. Both the instantaneous utility function for period $T_M$ ($u(c_{T_M})$) and the utility obtained when dying and bequeathing $x_{T_M+1}$ ($v(x_{T_M+1})$) are known and the model is solved for a discrete set of points on a grid. This gives us knowledge of $V_{T_M}$ at a subset of the points in the state space and allows us to approximate $V_{T_M}$ as $\hat{V}_{T_M}$ at all points. With this approximation in hand, we solve an approximation to problem (S.19) for period $T_M - 1$ and then, iteratively, for all preceding periods $t$:

$$V_t(b_{t-1}, A_{t-1}, s_{t-1}, \eta_{t-1}, \zeta_{t-1}) = \max_{(c_t, b_t, a_t, s_t, w_t, \eta_t) \in \mathcal{A}} (1-\beta)u(c_t) \tag{S.20}$$
$$-\frac{\beta}{k} \log \left( \pi_t E_t \left[ e^{-k\hat{V}_{t+1}(b_t, A_t, s_t, \eta_t, \zeta_t)} \right] + (1-\pi_t) E_t \left[ e^{-k(1-\beta)v(x_{t+1})} \right] \right).$$

Note that the only difference between the maximand in (S.20) and that in our household's problem is that the continuation value function is an approximate value function. We will now, briefly, discuss four features of the numerical procedure: i) the discretization of the continuous variables, ii) the integration of the value function, iii) the approximation method for evaluating the value function at points outside the discretized state space and iv) how the optimization is carried out.

**Discretization of the continuous variables.** We define a variable, total liquid wealth, which is the sum of bond and stock holdings at the start of a period. We define a grid of 54 points from \$0 to \$10m such that the gaps between successive grid points are smaller at lower levels of wealth, where the curvature of the value function will be greater. We define a grid of 36 points for annuity income from \$0 to \$800,000 such that the gaps between successive grid points are smaller at lower levels of income. Earnings are placed on a grid of 9 points each year using the procedure introduced by Tauchen (1986).

**Integration.** There are three risks facing households: mortality, earnings, and financial risks. The risks are independent. Realizations for the first of these are naturally discrete and integration involves a simple weighted average. For the latter two, we define a discrete set of possible realizations and integrate over outcomes using the Tauchen (1986) procedure.

**Approximation.** To evaluate the value function at points other than those in the discrete sub-set of points, we use linear interpolation in multiple dimensions.

**Optimization.** Every period households make up to four choices. They decide how much to consume, how much to save in each of bonds and the risky asset, whether to pay the participation cost (if they have not previously done so) and how much of an annuity income stream to purchase (in the period before retirement). Our problem is not globally concave, so our optimization of the household's decision problem cannot fully rely on local approaches. We therefore start by discretizing the choice variables. We define three grids on the unit interval, $S_a^t$, $S_s$, $S_c$, which will represent shares of available resources dedicated to purchases of annuities, portfolio shares in stocks and consumption respectively. All grids have equally spaced nodes from 0 to 1 (except that in periods other than period $T_R - 1$, the grid for annuities has only one element, 0, as no annuity purchase is possible in those periods). We evaluate the household's objective function at each combination of $(s^t_{a,i})_{i=1,\ldots,I^t}$ in $S^t_a$, $(s_{s,j})_{j=1,\ldots,J}$ in $S_s$, and $(s_{c,k})_{k=1,\ldots,K}$ in $S_c$. Defining the total available resources as $R_t = y_t + w_t$ (the sum of income and wealth) in time $t$, we set the annuity choice to $s^t_{a,i}R_t$, the level of saving in the risky asset set to $s_{s,j}(1 - s^t_{a,i})R_t$, the level of consumption as $s_{c,k}(1 - s_{s,j})(1 - s^t_{a,i})R_t$ and the savings in the bond as $(1 - s_{c,k})(1 - s_{s,j})(1 - s^t_{a,i})R_t$. We find the combination of the points that yields the maximum value to the household. This is our candidate optimal decision. We then do a further local search for the split between consumption and the bond using golden section search. Formally, taking the candidate maximum as indexed by $s^t_{a,i^*}$ in $S^t_a$, $s_{s,j^*}$ in $S_s$, $s_{c,k^*}$ in $S_c$, we look for the utility-maximizing split between consumption and the bond in the interval for consumption of $[s_{c,\max\{1,k^*-1\}}(1-s_{s,j^*})(1-s^t_{a,i^*})R, s_{c,\min\{K,k^*+1\}}(1-s_{s,j^*})(1-s^t_{a,i^*})R]$.

For points where individuals have not already paid the participation charge, we implement this procedure twice, once assuming they pay the charge, once assuming they do not. The maximum of these indicates the decision rule.

This procedure yields decision rules as a function of the vector of state variables $\boldsymbol{X}_t$: $\hat{c}(\boldsymbol{X}_t)_t, \hat{b}(\boldsymbol{X}_t)_t, \hat{s}(\boldsymbol{X}_t)_t, \hat{a}(\boldsymbol{X}_t)_t, \hat{\eta}(\boldsymbol{X}_t)_t$.

### S.4.2 Simulation of profiles

Once decision rules $\hat{c}(\boldsymbol{X}_t)_t, \hat{b}(\boldsymbol{X}_t)_t, \hat{s}(\boldsymbol{X}_t)_t, \hat{a}(\boldsymbol{X}_t)_t, \hat{\eta}(\boldsymbol{X}_t)_t$ are obtained, we simulate a data set for 3,000 individuals. We do this as follows:

1. Initial values for wealth are set to 0.
2. Earnings draws for the first period of economic life are drawn randomly for each individual. Using these values of the state variables and the decision rules we can obtain optimal behavior in the first period.
3. We draw an equity price shock to apply to any equity holdings held at the end of the period.
4. Optimal behavior, the rate of return and the inter-temporal budget constraint yield the state variables for period 2.
5. We repeat steps (2) to (4) to obtain optimal behavior and subsequent state variables for each age up to 100. In most time periods, individuals will have realizations of the continuous state variables that are off this grid. Our

approach here is to solve the individual's problem for decision rules as we do in the solution stage.

## S.5 The model extension to health risk

This section details the health-risk extension behind the sensitivity analysis in Section 4 and Appendix C.2 of the main text. Health shocks can in principle affect annuitization through two opposing forces. They make liquid wealth more valuable as insurance against medical expenses, but Medicaid means testing can also make annuities useful for sheltering resources from the asset test. Despite the possible role of health risk, our main quantitative results continue to hold to a large extent. We first introduce the health shock and the Medicaid floor in Section S.5.1, then describe the calibration of the health-expense process in Section S.5.2, and finally report how annuitization changes under the extension in Section S.5.3.

### S.5.1 Model extension

Agents in our baseline model were subject to risks over their earnings, rates of return, and survival. In this extension, we augment this baseline model with health risk. We introduce a stochastic health shock, denoted by $h$, which can afflict agents in retirement. The intertemporal budget constraint (equations (16) and (17) in the paper) for years after retirement in the extension is represented as:

$$c_t + qa_t + b_t + s_t + \mathbb{1}_{\eta_t=1}\mathbb{1}_{\eta_{t-1}=0}F = y_t + w_t, \tag{S.21}$$

$$\text{with:} \quad w_t = \max\{a_{T_R-1}\mathbb{1}_{t\geq T_R} + R^f b_{t-1} + R^s_t s_{t-1} - h_t, f\}, \tag{S.22}$$

where $h_t$ denotes health expenses in period $t$ and $f$ is a floor, representing a resource level, such that the government pays all health expenses that would bring resources below that level. In the context of the U.S., this formulation, in the spirit of De Nardi et al. (2016) and De Nardi et al. (2025), can be understood as approximating Medicaid in old age.[7] We assume that $h_t$ is *iid* log-normally distributed with a distribution that varies by age, with vectors of means and variances of $\{\mu_{h,t}\ \sigma^2_{h,t}\}_t$.

[7]Medicare covers those aged over 65, but does not fully insure all health expenses. Some expenses, most notably Long-Term Care, are not covered by Medicare and must be privately borne unless certain income and asset conditions are met.

### S.5.2 Data and calibration of health shock

Calibrating health expense risk in the US is complicated by the fact that there are many payers of health costs. Given that we include an approximation to public insurance in the model, our aim in calibrating the distribution of $h$ is to obtain a measure of health expenses that includes those shocks which end up being covered by public insurance. We rely on two recent careful empirical studies to calibrate the parameters of our age-dependent health process.

We proceed in three steps. First, we obtain mean health expenses *inclusive* of Medicaid expenditure from De Nardi et al. (2025) (Figure 5A).[8] That paper reports health expenses (inclusive of expenses covered by Medicaid) at selected ages. However, we need an estimate at all ages from 66. To obtain these, we regress the log of health expenses at the available selected ages on age and use the coefficients to predict average health expenses at all ages after retirement. The profile we obtain is plotted in Figure S.8, with the points from De Nardi et al. (2025) included on the graph.

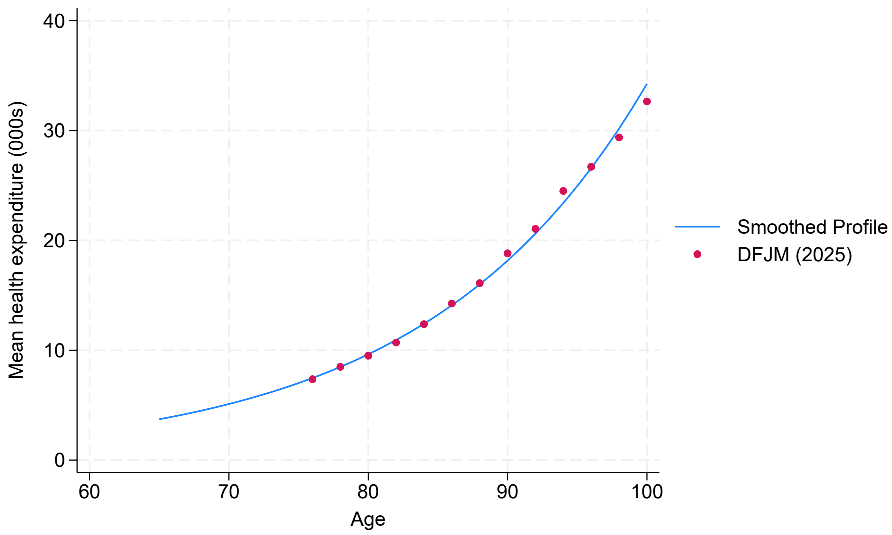



Figure S.8: Smoothed profile based on estimates from De Nardi et al. (2025).

This procedure gives us an estimate that we can use for the mean of health expenses, but not for its distribution. Our second step is to obtain estimates of the latter. We use the fact that Banks et al. (2019) provide (in Table 3 of their paper) estimates of the mean and five percentiles of health expenses for the elderly in the US. We use the relationship between each of these five percentiles and the mean expenses from that paper, and the estimates of the mean given in Figure S.8 (from

[8]As we build in the insurance against risk provided by Medicaid into the model, it is important that the measure of expenditure we record is inclusive of public expenditures.

De Nardi et al., 2025) to obtain estimates of percentiles of expenses at each age.[9]

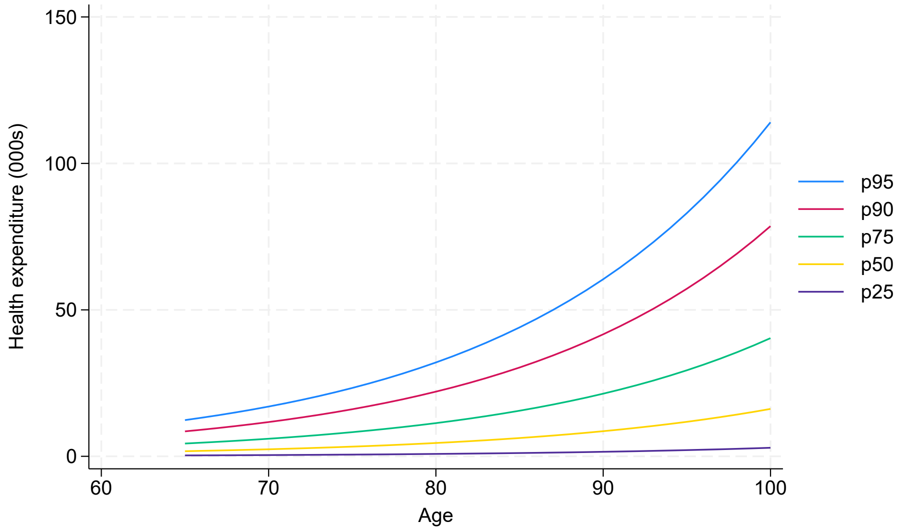



Figure S.9: Age profile of health spending percentiles calculated combining estimates from Banks et al. (2019) and De Nardi et al. (2025).

In a third and final step we estimate the mean and variance of $h_t$ by finding estimates of $\mu_{h,t}$ and $\sigma^2_{h,t}$ that minimize the squared distances from the percentiles they imply and the estimated means and percentiles shown in Figures S.8 and S.9.

One final object needs to be set: the floor ($f$), below which level of resources the government funds all medical costs. The actual floors used in the US vary across states, and the institutional setting is further complicated by a consideration of which assets are included as 'countable' assets for the housing test. We make two remarks here. The first relates to wealth which has been annuitized. Income flows from such wealth (i.e., the annuity payment) are included as resources in the year which they are received and are countable towards the asset test. Future income flows from the same annuity contract, however, can be excluded.[10] If the wealth had *not* been annuitized, on the other hand, the entire stock could be considered as 'countable' and considered available to pay for financial health shocks. Thus, annuitization can be a device that allows individuals to shelter their wealth from the Medicaid asset test. Reflecting this institutional feature, we include annuity *income* in the resources which are countable towards the asset test, but exclude future annuity income flows.

[9]More specifically, and using the tenth percentile, as an example, we calculate the ratio of the 10th percentile to the mean using the figures in Banks et al. (2019) (denoted by BBLS): $\hat{r}^{BBLS}_{10} = \frac{p^{BBLS2019}_{10}}{\mu^{BBLS2019}}$. We then obtain an estimate of the tenth percentile of medical spending at (for example) age 70 by multiplying our estimate of the mean from De Nardi et al. (2025) by $\hat{r}^{BBLS}_{10}$.

[10]This happens if individuals in the US purchase a 'Medicaid Compliant Annuity'.

Our second remark concerns the treatment of housing wealth in the Medicaid means test. In principle housing wealth can, for singles (who we model), be included as a 'countable asset' in the measure of wealth considered in the Medicaid asset test – i.e., there is an expectation that individuals will sell their house before claiming Medicaid. However, in practice, its value can be excluded in certain circumstances. Achou (2023) shows that close to a third of all singles in receipt of Medicaid over the age of 70 are homeowners. To accommodate the fact that not all wealth is 'counted' and that housing wealth can be exempt, we set a baseline floor at average income $\bar{y}$, but show selected results with a different floor below.[11]

### S.5.3 The role of health risk on annuitization behavior

We present two sets of results. First, we show how the introduction of health shocks affects annuitization behavior when the calibrated parameters are held at their baseline values, under both RS and additive specifications. We do this for two different assumptions about the resource floor $f$. Second, for one of these floors, we recalibrate the parameters to target the same objects as in baseline, and show how the parameters change.

#### S.5.3.1 Annuitization behavior holding parameters at baseline calibration

Figure S.10 shows the proportion of households annuitizing in the case where we introduce health risk but keep the parameters at their baseline levels. We show results for two asset-test floors. The first pair of bars has a floor of 20% of average income ('Floor 1'), while the second pair has a floor of 100% of average income ('Floor 2').

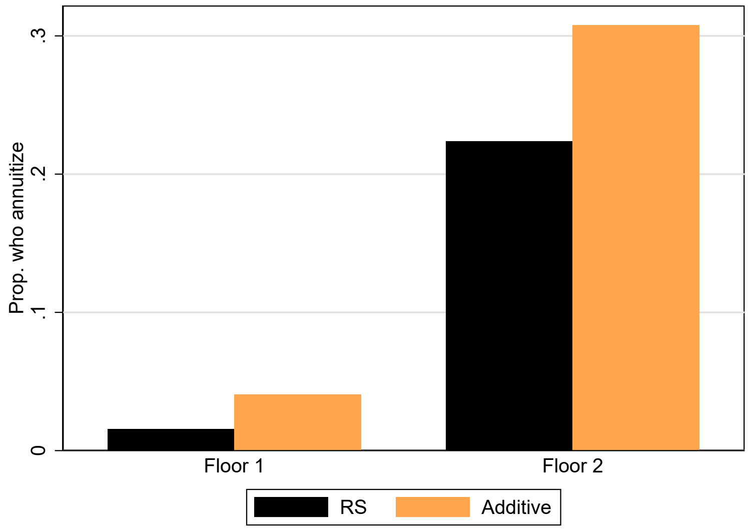
We take three lessons from this graph:

1. Health risk reduces the demand for annuities in the case where there is limited public insurance against health shocks. This can be seen when looking at the bars on the left of the graph, which show that the proportion of agents with annuities falls below the 5% level, which was the target share. The reason for this is that, when faced with the prospect of substantial shocks to resources, having liquid wealth can insure against the lowest levels of consumption.[12]

[11]We assume that the health shock is realized at the very start of the period, just before mortality is realized. Agents only get access to their Medicaid floor if they survive. If they die after the shock, any assets they have are assumed to be available to pay for the health expense shock.

[12]An offsetting force is that households accumulate more wealth to insure against such risk, which *increases* annuitization, but this force is not quantitatively large enough to outweigh the desire to hold liquidity.

Figure S.10: Annuitization under baseline calibration, by Medicaid resource floor.




2. When the Medicaid floor is set higher, at 100% of average income, more wealth is potentially subject to the asset test, and the fact that annuities are a useful device to shelter wealth from the asset-test implies that annuitization actually *increases* once we include health risk.

3. In both cases, annuitization rates are higher under the additive specification than under the RS specification. This will have implications for the calibrated parameters below.

#### S.5.3.2 Recalibrated model

The extended model with health expense risk is calibrated to the same targets as our baseline model. We consider the case of a floor of 100% of average income ('Floor 2' above).

Table S.3: Comparing parameter calibration in the baseline and health risk models.

| Parameter | Baseline | | Health Risk Extension | |
|---|---|---|---|---|
| | RS | Additive | RS | Additive |
| Risk aversion parameter, $k$ | 1.02 | 0.00† | 1.06 | 0.00† |
| Life-death utility gap, $u_l$ | 4.3 | 15.5 | 4.1 | 16.2 |
| Discount factor, $\beta$ | 0.975 | 0.953 | 0.971 | 0.948 |
| Annuity admin. load, $\delta$ | 10%† | 23% | 10%† | 29% |
| Participation cost, $F$ | 111% of $\bar{y}$ | 123% of $\bar{y}$ | 66% of $\bar{y}$ | 79% of $\bar{y}$ |

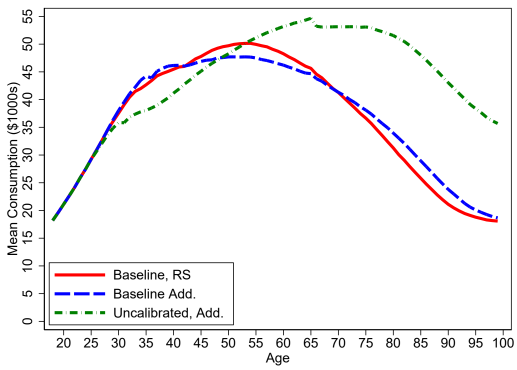
*Notes:* One unit of consumption is equal to $\bar{y}$. Quantities indicated by a † are imposed rather than estimated.

Table S.3 compares the calibrated parameters under the baseline and extended models. Patience is lower in both the RS and additive specifications, as there is an additional reason to hold wealth. The life-death utility gap is modestly impacted in both specifications. The cost of participating in the risky asset market falls significantly in both the RS and additive models, reflecting that highest wealth levels at old age (achieved by the high returns of risky assets) are at risk of being 'taxed' away by the Medicaid asset test. The administrative annuity load, constrained to 10% in the RS model, increases from 23% in the calibrated additive model without health risk to 29% in the calibrated additive model with health risk. That is, the wedge in the administrative load required to match annuitization behavior, in the additive and RS specifications, is larger in the extended model than in our baseline.

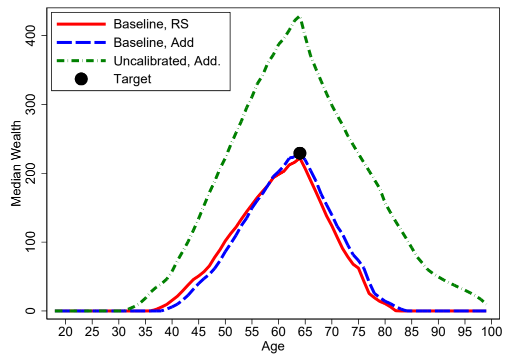
For completeness, Figure S.11 shows the life-cycle profiles for the calibrated model with health expense risk. This is analogous to Figure 1 in the paper. By construction the target values are similar, and in general the paths do not diverge much from those in the baseline model.

Figure S.11: Life-cycle profiles for the calibration, with health.

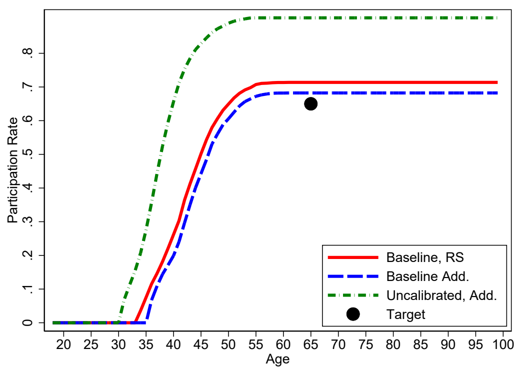
(a) Consumption



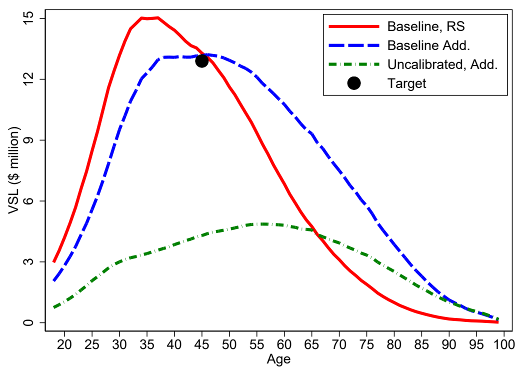

(b) Median wealth




(c) Stock market participation




(d) VSL



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